سوزان باقری / دهم دختر C / تکلیف ۲۱

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$1!=1$

$1!+3!=7$

$1!+3!+5!=127$

$1!+3!+5!+7!=5167$

$(2!+4!+6!+\dots+26!)$ — از اینجا یکان صفر است

$\underbrace{2+4}_{6}$ ، (7)

$6\times7=42 \Rightarrow 2$

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۲- الف)

$$\sqrt[4]{\frac{1}{4+\sqrt7}}\sqrt[4]{(1+\sqrt7)^2}=\sqrt[4]{(4+\sqrt7)^{-1}(1+7+2\sqrt7)}=\sqrt[4]{2}$$

$$\frac{8+2\sqrt7}{4+\sqrt7}=\frac{2\cancel{(4+\sqrt7)}}{\cancel{(4+\sqrt7)}}=2$$

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ب)

$$\frac{\cancel{\sqrt2+\sqrt5}}{\sqrt2\cancel{(\sqrt5+\sqrt2)}}=\frac{1}{\sqrt2}$$

$$\frac{\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}}{\sqrt2}\times\frac{\sqrt2}{\sqrt2}=\frac{\sqrt{6-2\sqrt5}-\sqrt{6+2\sqrt5}}{2}$$

$$\frac{\sqrt{(\sqrt5+1)^2}-\sqrt{(\sqrt5-1)^2}}{2}=\frac{|\sqrt5+1|-|\sqrt5-1|}{2}=\frac{-2}{2}=-1$$

۳- الف)

$$\frac{\overset{2\sqrt2}{\sqrt8}+\overset{3\sqrt3}{\sqrt{27}}}{5-\underset{\sqrt2\times\sqrt3}{\sqrt6}}-2(\sqrt3-1)^{-1}=\frac{(2\sqrt2+3\sqrt3)(\sqrt2+\sqrt3)}{(\sqrt2^2+\sqrt3^2-\sqrt3\times\sqrt2)(\sqrt2+\sqrt3)}-\frac{2(\sqrt3+1)}{2}$$

$\sqrt2^2+\sqrt3^2 \leftarrow 5$ ، فیل وفنجون ، $\frac{2}{\sqrt3-1}$

$$\Rightarrow\sqrt2+\sqrt3-\sqrt3-1=\boxed{\sqrt2-1}$$

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ب)

$$\frac{3\sqrt3-1}{4+\sqrt3}+\frac{1}{2-\sqrt3}=\sqrt3-1+2+\sqrt3=1+2\sqrt3$$

$$\frac{3\sqrt3-1}{4+\sqrt3}\times\frac{4-\sqrt3}{4-\sqrt3}=\frac{12\sqrt3-9-4+\sqrt3}{16-3}=\frac{13\sqrt3-13}{13}=\frac{13(\sqrt3-1)}{13}=\sqrt3-1$$

$$\frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3}=\frac{2+\sqrt3}{4-3}=\frac{2+\sqrt3}{1}=2+\sqrt3$$

۴- الف)

$$\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}=\frac{\sqrt{(\sqrt3+1)^2+(\sqrt3-1)^2}}{\sqrt{\sqrt3-\sqrt2}}=\frac{\sqrt{2\sqrt3+2\sqrt2}}{\sqrt{\sqrt3-\sqrt2}}\xrightarrow{\text{ضرب در مزدوج}}2+\sqrt6-2=\sqrt6$$

ب)

$$\sqrt[4]{\sqrt[3]{2^8}}\times\sqrt[4]{162}\times\sqrt{4\sqrt[6]{2}}$$

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$$\sqrt[12]{2^8}\times\sqrt[4]{162}\times\sqrt[12]{4^6\times2}$$

$$\sqrt[12]{2^8}\times\sqrt[12]{(162)^3}\times\sqrt[12]{4^6\times2}=\sqrt[12]{2^8\times3^{12}\times2^3\times2^{13}}=\sqrt[12]{3^{12}\times2^{24}}=12$$

$(3^4\times2)^3=3^{12}\times2^3$ ، $(2^2)^6=2^{12}$

۵-

$$\frac{3^x(1+3+3^2+3^3+3^4+3^5)}{2^x(2^{-2}+2^{-1}+2^0+2^1+2^2+2^3)} = \frac{3^x((3^6-1)\div 2)}{2^{x-2}\left(\frac{2^6-1}{1}\right)} = \frac{3^{x-2}}{3^{x-2}}$$

$$\boxed{x=2}$$

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$(a^2+b^2+2ab)^2 = t(2-\sqrt{3})$

$\Rightarrow 24+8 \ldots = 32-8\sqrt{12}$

$8\sqrt{12}$

جواب در صفحه بعد

۷-

$$((a+\frac{1}{a})^2-2)^2 = (a^2+\frac{1}{a^2}+2-2)^2 = (a^2+\frac{1}{a^2})^2 = (2-\sqrt{3}+\frac{1}{2-\sqrt{3}})^2 = (2-\sqrt{3}+2+\sqrt{3})^2$$

$$4^2 = (2^2)^2 = 2^4$$

$$\boxed{t=4}$$

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-۸

$A = \sqrt[5]{4\sqrt[3]{16}}\left(\frac{1}{2}\right)^{-\frac{4}{3}} =$

$2^{\frac{2}{5}} \times 2^{\frac{4}{15}} \times 2^{\frac{4}{3}} = 2^{\frac{30}{15}} = 2^{2}$ $\qquad (2\times 4)^{-\frac{1}{3}} = 2^{-1} = \frac{1}{2}$

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-۹

$\sqrt[7]{\frac{1}{a^{14}}} = 27 \qquad \frac{1}{a^{2}} = 27 \Longrightarrow a = \frac{\sqrt{3}}{9}$

$\frac{\frac{1}{a} - 3}{1+\sqrt{3}} = \frac{\frac{9}{\sqrt{3}} - 3}{1+\sqrt{3}} = \frac{\frac{9\sqrt{3}}{3} - \frac{9}{3}}{1+\sqrt{3}} = \frac{\frac{9}{3}(\sqrt{3}-1)}{1+\sqrt{3}} =$

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$\xRightarrow{\text{ضرب در مزدوج}} \frac{3(\sqrt{3}-1)^{2}}{2} = \frac{12-6\sqrt{3}}{2} = \boxed{6-3\sqrt{3}}$

-۱۰

$x + a - x + 4 = 2\sqrt{a+x} + \sqrt{x-4}$

$\frac{a+4}{2} = \sqrt{a+x} + \sqrt{x-4} \Longrightarrow \underbrace{\cancel{\sqrt{x+a}+\sqrt{x-4}}}_{\frac{a+4}{2}} - 2 = \frac{a+4-4}{2} = \frac{a}{2}$

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-۶

$(a^{2}+b^{2}-2ab)^{2}(a^{2}+b^{2}+2ab)^{2} = t(2-\sqrt{3})$

$\left(\left(\sqrt[4]{\sqrt{6}-2}\right)^{4} + \left(\sqrt[4]{\sqrt{6}+2}\right)^{4} - 2\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)} =\right.$

$32 - 16\sqrt{3} = t(2-\sqrt{3}) \Longrightarrow \boxed{t = 16}$

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