۱-

$1! = 1$

$1! + 3! = 7$

$1! + 3! + 5! = 127$

$1! + 3! + 5! + 7! = 5167$

$\left\{ 7 \right.$

$(2! + 4! + 6! + \dots + 26!)$ — از اینجا یکان صفر میشه

$\underbrace{2+4}_{6}$

$6 \times 7 = 42 \Rightarrow 2$

۲- الف)

$$\sqrt[4]{\frac{1}{4+\sqrt{7}}}\ \sqrt[4]{(1+\sqrt{7})^2} = \sqrt[4]{(4+\sqrt{7})^{-1}(\underbrace{1+7}_{8}+2\sqrt{7})} = \sqrt[4]{2}$$

$$\frac{8+2\sqrt{7}}{4+\sqrt{7}} = \frac{2(4+\sqrt{7})}{(4+\sqrt{7})} = 2$$

ب)

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}(\sqrt{5}+\sqrt{2})} = \frac{1}{\sqrt{2}} \qquad \frac{\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6-2\sqrt{5}}-\sqrt{6+2\sqrt{5}}}{2}$$

$$\frac{\sqrt{(\sqrt{5}+1)^2}-\sqrt{(\sqrt{5}-1)^2}}{2} = \frac{|\sqrt{5}+1|-|\sqrt{5}-1|}{2} = \frac{-2}{2} = -1$$

۳- الف)

$$\frac{\overset{2\sqrt{2}}{\sqrt{8}}+\overset{3\sqrt{3}}{\sqrt{27}}}{\underset{\sqrt{2}^2+\sqrt{3}^2}{5}-\underset{\sqrt{2}\times\sqrt{3}}{\sqrt{6}}} - 2(\sqrt{3}-1)^{-1} = \frac{(2\sqrt{2}+3\sqrt{3})(\sqrt{2}+\sqrt{3})}{(\sqrt{2}^2+\sqrt{3}^2-\sqrt{3}\times\sqrt{2})(\sqrt{2}+\sqrt{3})} - \frac{2(\sqrt{3}+1)}{2}$$

فیل وفنجون

$\frac{2}{\sqrt{3}-1}$

$$\Rightarrow \sqrt{2}+\sqrt{3}-\sqrt{3}-1 = \boxed{\sqrt{2}-1}$$

ب)

$$\frac{3\sqrt{3}-1}{4+\sqrt{3}} + \frac{1}{2-\sqrt{3}} = \sqrt{3}-1+2+\sqrt{3} = 1+2\sqrt{3}$$

$$\frac{3\sqrt{3}-1}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{12\sqrt{3}-9-4+\sqrt{3}}{16-3} = \frac{13\sqrt{3}-13}{13} = \frac{13(\sqrt{3}-1)}{13} = \sqrt{3}-1$$

$$\frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = \frac{2+\sqrt{3}}{1} = 2+\sqrt{3}$$

۴- الف)

$$\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} = \frac{\sqrt{(\sqrt{3}+1)^2+(\sqrt{3}-1)^2}}{\sqrt{\sqrt{3}-\sqrt{2}}} = \frac{\sqrt{2\sqrt{3}+2\sqrt{2}}}{\sqrt{\sqrt{3}-\sqrt{2}}} \xRightarrow{\text{ضرب در مزدوج}} 2+\sqrt{6}-2 = \sqrt{6}$$

ب)

$$\sqrt[4]{\sqrt[3]{3^8}} \times \sqrt[4]{162} \times \sqrt{4\sqrt[6]{2}}$$

$$\sqrt[12]{3^8} \times \sqrt[4]{162} \times \sqrt[12]{4^6 \times 2}$$

$$\sqrt[12]{3^8} \times \sqrt[12]{(162)^3} \times \sqrt[12]{4^6 \times 2} = \sqrt[12]{3^8 \times 3^{12} \times 2^3 \times 2^{13}} = \sqrt[12]{3^{12} \times 2^{24}} = 12$$

$(3^4 \times 2)^3 = 3^{12} \times 2^3$

$(2^2)^6 = 2^{12}$

۵-

$$\frac{3^x(1+3+3^2+3^3+3^4+3^5)}{2^x(2^{-2}+2^{-1}+2^0+2^1+2^2+2^3)} = \frac{3^x\left((3^6-1)\div 2\right)}{2^{x-2}\left(\frac{2^6-1}{1}\right)} = \frac{3^{x-2}}{3^{x-2}}$$

$$\boxed{x=2}$$

۶-

$$\cancel{(a^2+b^2-2ab)^2}\,(a^2+b^2+2ab)^2 = t(2-\sqrt3)$$

$$\cancel{4\left(\sqrt{\sqrt6+2}+\sqrt{\sqrt6-2}-2\sqrt{20}\right)\left(\sqrt{\sqrt6+2}+\sqrt{\sqrt6-2}+2\sqrt{\sqrt{20}}\right)^2} =$$

$$\cancel{\sqrt{\sqrt6+2}+\sqrt{\sqrt6-2}-2\sqrt2 \Rightarrow \sqrt6+2+\sqrt6-2+2\sqrt{\ }-4\sqrt2}$$

$$\cancel{\Rightarrow 24+8-4(2\sqrt6+2\sqrt4) = 32-8\sqrt{32}}$$

$$\cancel{4\sqrt{12}}$$

$$\cancel{8\sqrt{12}}$$

جواب در صفحه بعد

۷-

$$\left((a+\frac1a)^2-2\right)^2 = \left(a^2+\frac{1}{a^2}+2-2\right)^2 = \left(a^2+\frac{1}{a^2}\right)^2 = \left(2-\sqrt3+\frac{1}{2-\sqrt3}\right)^2 = \left(2-\sqrt3+2+\sqrt3\right)^2 =$$

$$4^2 = (2^2)^2 = 2^4$$

$$\boxed{t=4}$$

۸-

$$A = \sqrt[5]{4\sqrt[3]{16}}\left(\frac{1}{2}\right)^{-\frac{4}{3}} = \qquad (2\times 4)^{-\frac{1}{3}} = 2^{-1} = \frac{1}{2}$$

$$2^{\frac{2}{5}} \times 2^{\frac{4}{15}} \times 2^{\frac{4}{3}} = 2^{\frac{30}{15}} = 2^{2}$$

۹-

$$\sqrt[7]{\frac{1}{a^{14}}} = 27 \qquad \frac{1}{a^{2}} = 27 \Longrightarrow a = \frac{\sqrt{3}}{9}$$

$$\frac{\frac{1}{a}-3}{1+\sqrt{3}} = \frac{\frac{9}{\sqrt{3}}-3}{1+\sqrt{3}} = \frac{\frac{9\sqrt{3}}{3}-\frac{9}{3}}{1+\sqrt{3}} = \frac{\frac{9}{3}(\sqrt{3}-1)}{1+\sqrt{3}} =$$

ضرب در مزدوج $\Longrightarrow \dfrac{3(\sqrt{3}-1)^{2}}{2} = \dfrac{12-6\sqrt{3}}{2} = \boxed{6-3\sqrt{3}}$

۱۰-

$$x + a - x + 4 = 2\sqrt{a+x} + \sqrt{x-4}$$

$$\frac{a+4}{2} = \sqrt{a+x} + \sqrt{x-4} \Longrightarrow \cancel{\sqrt{x+a}+\sqrt{x-4}} - 2 = \frac{a+4-4}{2} =$$

$$\frac{a+4}{2} \qquad \frac{a}{2}$$

۶-

$$(a^{2}+b^{2}-2ab)^{2}(a^{2}+b^{2}+2ab)^{2} = t(2-\sqrt{3})$$

$$\left(\left(\sqrt[4]{\sqrt{6}-2}\right)^{4} + \left(\sqrt[4]{\sqrt{6}+2}\right)^{4} - 2\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)} =$$

$$32-16\sqrt{3} = t(2-\sqrt{3}) \Longrightarrow \boxed{t = 16}$$