

$$1! = 1$$

$$1! + 1! = 2$$

$$1! + 1! + 1! = 3$$

$$1! + 1! + 1! + 1! = 4$$

$$2! = 2$$

$$2! + 1! = 3$$

$$2! + 1! + 1! = 4$$

$$2! + 1! + 1! + 1! = 5$$

$$(- \dots - V)(- \dots - A) = \Delta \Rightarrow \text{بين قوسين} = 4$$

$$\text{الف) } (K + \sqrt{V})^{\frac{1}{2}} \times (1 + \sqrt{V})^{\frac{1}{2}} = \frac{(1 + \sqrt{V})^{\frac{1}{2}}}{(K + \sqrt{V})^{\frac{1}{2}}} \times K \sqrt{\frac{(1 + \sqrt{V})^{\frac{1}{2}}}{(K + \sqrt{V})^{\frac{1}{2}}}} = K \sqrt{V} \quad \text{مسألة (2)}$$

$$(1 + \sqrt{V})^2 = 1 + V + 2\sqrt{V} = 1 + 2\sqrt{V} = 2(\sqrt{V} + 1)$$

$$\text{ب) } \frac{\sqrt{2 + \sqrt{5}}}{\sqrt{2}(\sqrt{5} + \sqrt{2})} = \frac{1}{\sqrt{2}} \Rightarrow \frac{\sqrt{10 - \sqrt{5}} - \sqrt{10 + \sqrt{5}}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{4 - 10\sqrt{5}} - \sqrt{4 + 10\sqrt{5}}}{2}$$

$$\left. \begin{array}{l} \sqrt{10} \\ \sqrt{2} \end{array} \right\} \Rightarrow \frac{\sqrt{10 + 1}^2 - \sqrt{10 - 1}^2}{2} = \frac{|\sqrt{10 + 1}| - |\sqrt{10 - 1}|}{2} = \frac{-1}{2} = -\frac{1}{2}$$

مسألة (3)

$$\text{الف) } \frac{\sqrt{1 + \sqrt{2}}}{\omega - \sqrt{4}} = \left(\sqrt{\frac{1}{9}} - 1 \right)^{-1} = \frac{2\sqrt{2} + 1\sqrt{2}}{12\sqrt{2} - \omega - \sqrt{4}\sqrt{2}} = \frac{-1}{\sqrt{2} - 1}$$

$$= \frac{(2\sqrt{2} + 1\sqrt{2})(\sqrt{2} + \sqrt{2})}{[(\sqrt{2} + \sqrt{2})(\sqrt{2} + \sqrt{2}) - \sqrt{2} + \sqrt{2}]} = \frac{2\sqrt{2} + 1\sqrt{2}}{2\sqrt{2} - 1} = \sqrt{2} - 1$$

$$\text{ب) } \frac{\sqrt{2V - 1}}{K + \sqrt{K}} + (K - \sqrt{K})^{-1} = \left(\frac{K\sqrt{K} - 1}{K + \sqrt{K}} \times \frac{\sqrt{K} - 1}{\sqrt{K} - 1} \right) + \left(\frac{1}{K - \sqrt{K}} + \frac{K + \sqrt{K}}{K + \sqrt{K}} \right)$$

$$= \frac{K\sqrt{K} - 1}{K + \sqrt{K}} + \frac{K + \sqrt{K}}{1} = \sqrt{K} - 1 + K + \sqrt{K} = 2\sqrt{K} + K - 1$$

مسألة (4)

$$\text{الف) } \frac{\sqrt{1 + \sqrt{K}} + \sqrt{\sqrt{K} - 1}}{\sqrt{\sqrt{K} - \sqrt{K}}} = \frac{\sqrt{1 + \sqrt{K}} + \sqrt{\sqrt{K} - 1}}{\sqrt{\sqrt{K} - \sqrt{K}}} \times \frac{\sqrt{\sqrt{K} + \sqrt{K}}}{\sqrt{\sqrt{K} + \sqrt{K}}}$$

$$= \left(\sqrt{1 + \sqrt{K}} + \sqrt{\sqrt{K} - 1} \right) (\sqrt{\sqrt{K} - \sqrt{K}}) = \sqrt{[(1 + \sqrt{K}) + (\sqrt{K} - 1)] (\sqrt{\sqrt{K} - \sqrt{K}})^2}$$

$$= \sqrt{(K\sqrt{K} - K\sqrt{K}) (\sqrt{K} - \sqrt{K})} = (\sqrt{K} - \sqrt{K}) \sqrt{K} = \sqrt{K} (\sqrt{K} - \sqrt{K}) = \sqrt{4 - K}$$

$$\text{ب) } \frac{K\sqrt{K}}{\sqrt{K}} \times \frac{K}{\sqrt{14K}} \times \sqrt{K\sqrt{K}} = \frac{K\sqrt{K}}{\sqrt{K}} \times \frac{K}{\sqrt{14K}} \times \sqrt{K\sqrt{K}} = \frac{K}{\sqrt{14}} \times \frac{K}{\sqrt{K}} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{K}{2}$$

سؤال ٩

$$\frac{10^9 (1 + 10 + 10^2 + \dots + 10^9)}{10^{9-p} (1 + 10 + 10^2 + \dots + 10^9)} = \frac{10^9 \left(\frac{10^{10} - 1}{10 - 1} \right)}{10^{9-p} \left(\frac{10^{10} - 1}{10 - 1} \right)} = \frac{10^{9-p}}{10^{9-p}} = 1 \Rightarrow 9 = p$$

سؤال ٩

$$a = \sqrt[4]{4-p}, \quad b = \sqrt[4]{4+p} \Rightarrow (a^4 + b^4 - 2ab) (a^4 + b^4 + 2ab) = 4(4 - \sqrt{p})$$

$$(a^4 + b^4)^2 - 4a^4 b^4 = (a^4 + b^4 + 2ab)(a^4 + b^4 - 2ab) = 4(4 - \sqrt{p})$$

$$10^4 - 14\sqrt{p} = 4(4 - \sqrt{p}) \Rightarrow 14(4 - \sqrt{p}) = 4(4 - \sqrt{p}) \Rightarrow 14 = 4 \Rightarrow 14$$

$$= (\sqrt[4]{4-p})^4 + (\sqrt[4]{4+p})^4 - 2\sqrt{(4-p)(4+p)} = 4 - p + 4 + p - 2\sqrt{16 - p^2} = 8 - 2\sqrt{16 - p^2} = 4 - 14\sqrt{p}$$

$$= 2\sqrt{4} - 2\sqrt{p} \Rightarrow 2\sqrt{4 - p^2} = 4 - 14\sqrt{p} \Rightarrow 16 - 4p^2 = 16 - 14\sqrt{p} \Rightarrow 14\sqrt{p} = 4p^2 \Rightarrow 14 = 4p^2 \Rightarrow p^2 = \frac{14}{4} \Rightarrow p = \frac{\sqrt{14}}{2}$$

سؤال ٧

$$a = \sqrt[4]{1 - 4\sqrt{p}}, \quad b = \sqrt[4]{1 + 4\sqrt{p}} \Rightarrow (a + \frac{1}{a} + \sqrt{p})^4 (a + \frac{1}{a} - \sqrt{p})^4 = 16$$

$$= \left((a + \frac{1}{a})^2 - p \right)^4 = (a^2 + \frac{1}{a^2} + 2 - p)^4 = 16$$

$$(1 - \sqrt{p} + \frac{1}{1 - \sqrt{p}})^4 = (1 - \sqrt{p} + 1 + \sqrt{p})^4 = 4^4 = 256 \Rightarrow 16 = 256 \Rightarrow 16 = 256 \Rightarrow 16 = 256$$

سؤال ٨

$$A = \sqrt[4]{16\sqrt{14}} \left(\frac{1}{4} \right)^{\frac{4}{16}} = 16^{\frac{1}{4}} \times \frac{1}{4} = 2 \times \frac{1}{4} = \frac{1}{2}$$

$$(1/A)^{\frac{1}{4}} = (1^{\frac{1}{4}})^{\frac{1}{4}} = \left(\frac{1}{1^{\frac{1}{4}}} \right)^{\frac{1}{4}} = \sqrt[4]{\frac{1}{1^{\frac{1}{4}}}} = \frac{1}{1^{\frac{1}{4}}} = \frac{1}{1}$$

سؤال ٩

$$\frac{\sqrt[4]{a}}{\sqrt[4]{a^4}} = 16 \Rightarrow \sqrt[4]{\frac{1}{a^4}} = 16 \Rightarrow \frac{1}{a^4} = 16 \Rightarrow a^4 = \frac{1}{16} \Rightarrow a = \frac{1}{2}$$

سؤال ١٠

$$\sqrt{x+a} - \sqrt{x-a} = 2 \Rightarrow (\sqrt{x+a} - \sqrt{x-a})(\sqrt{x+a} + \sqrt{x-a}) = 2(\sqrt{x+a} + \sqrt{x-a})$$

$$x+a - x-a = 2\sqrt{x+a} + 2\sqrt{x-a} \Rightarrow \frac{a+x}{2} = \sqrt{x+a} + \sqrt{x-a}$$

$$\sqrt{x+a} + \sqrt{x-a} = 2 \Rightarrow \frac{a+x}{2} = 2 \Rightarrow \frac{a+x}{2} = 2 \Rightarrow a+x = 4 \Rightarrow x = 4-a$$