نام و نام خانوادگی ...نرگس اسلامی نژاد...... پاسخنامه تشریحی تکلیف شماره ..۲۱.. کلاس دهم [illegible]........

۱

$(1!+3!+5!+\cdots+25!)$ ، $\underbrace{\cdots+25!}_{0}$

1 ، $1\times2\times3=6$ ، $6+1=7$

$(2!+4!+6!+\cdots+26!)$ ، $\underbrace{\cdots+26!}_{0}$

2 ، $1\times2\times3\times4=24 \to 4$

$4+2=6$

$7\times6=4\boxed{2}$

۲

الف)

$\sqrt[4]{(4+\sqrt7)^{-1}}\times\sqrt{1+\sqrt7}$

$\sqrt[4]{\frac{1}{4+\sqrt7}}\times\sqrt[4]{(1+\sqrt7)^2}$

$\sqrt[4]{\frac{(1+\sqrt7)^2}{4+\sqrt7}}=\sqrt[4]{\frac{1+7+2\sqrt7}{4+\sqrt7}}=\sqrt[4]{\frac{8+2\sqrt7}{4+\sqrt7}}=\sqrt[4]{\frac{2(\cancel{4+\sqrt7})}{\cancel{4+\sqrt7}}}=\boxed{\sqrt[4]{2}}$

ب)

$\left(\frac{\sqrt2+\sqrt{10}}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)$

$\frac{\sqrt2+\sqrt{10}}{\sqrt{10}+2}\times\frac{\sqrt{10}-2}{\sqrt{10}-2}=\frac{3\sqrt2}{6}=\frac{\sqrt2}{2}$

$\frac{\sqrt2}{2}\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)$

$\frac{\sqrt{2(3-\sqrt5)}}{2}=\frac{\sqrt{6-2\sqrt5}}{2}=\frac{\sqrt{(\sqrt5-1)^2}}{2}=\frac{\sqrt5-1}{2}$

$\frac{\sqrt{2(3+\sqrt5)}}{2}=\frac{\sqrt{6+2\sqrt5}}{2}=\frac{\sqrt{(\sqrt5+1)^2}}{2}=\frac{\sqrt5+1}{2}$

$\frac{\cancel{\sqrt5}-1}{2}-\frac{\cancel{\sqrt5}+1}{2}=\frac{-2}{2}=\boxed{-1}$

۳

الف)

$\frac{\overset{2\sqrt2}{\sqrt8}+\overset{3\sqrt3}{\sqrt{27}}}{5-\sqrt6}\times\frac{5+\sqrt6}{5+\sqrt6}=\frac{19\sqrt3+19\sqrt2}{19}=\sqrt3+\sqrt2$

$(\sqrt[4]{9}-1)^{-1}=\frac{1}{\sqrt3-1}\times\frac{\sqrt3+1}{\sqrt3+1}=\frac{\sqrt3+1}{3-1}=\frac{\sqrt3+1}{2}\xrightarrow{\times2}\sqrt3+1$

$\sqrt[4]{3^2}=\sqrt3$

$\sqrt3+\sqrt2-\sqrt3-1=\boxed{\sqrt2-1}$

ب)

$\frac{\overset{3\sqrt3}{\sqrt{27}}-1}{4+\sqrt3}\times\frac{4-\sqrt3}{4-\sqrt3}=\frac{\cancel{13}\sqrt3-\cancel{13}}{\cancel{13}}=\sqrt3-1$

$\frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3}=\frac{2+\sqrt3}{1}=2+\sqrt3$

$\sqrt3-1+2+\sqrt3=\boxed{2\sqrt3+1}$

۴

الف)

$\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}\xrightarrow{\text{توان ۲}}1+\sqrt3+\sqrt3-1+2\sqrt{\underbrace{(\sqrt3+1)(\sqrt3-1)}_{2}}=2\sqrt3+2\sqrt2=2(\sqrt3+\sqrt2)$

$\to\sqrt3-\sqrt2$

$\to2\,\frac{\sqrt3+\sqrt2}{\sqrt3-\sqrt2}\times\frac{\sqrt3+\sqrt2}{\sqrt3+\sqrt2}=\frac{2(3+2+2\sqrt6)}{1}=2(5+2\sqrt6)=10+4\sqrt6\xrightarrow{\text{جذر}}\sqrt{10+4\sqrt6}\to\sqrt{(\sqrt6+2)^2}=\sqrt6+2\xrightarrow{-2}\boxed{\sqrt6}$

ب)

$\sqrt[12]{2^8}\times\underbrace{3\times\sqrt[12]{2^3}}\times\sqrt[12]{2\times2^{12}}=\sqrt[12]{2^8\times2^3\times2\times2^{12}}=\sqrt[12]{2^{24}}=2^2=4\times3=\boxed{12}$

$162=81\times2=\sqrt[4]{3^4\times2}=3\times\sqrt[12]{2^3}$

۵

$\frac{3^x(1+3^1+3^2+3^3+3^4+3^5)}{2^{x-2}(1+2^1+2^2+2^3+2^4+2^5)}=\frac{3^x\times364}{2^x\times2^{-2}\times63}=\frac{52}{1}$

$3^x\times1\times\overset{28}{\cancel{364}}=2^x\times\frac{1}{\cancel{4}}\times63\times\cancel{52}$

$3^x\times2^2\times\cancel{7}=2^x\times3^2\times\cancel{7}\to\boxed{x=2}$

6

$((a-b)^2)^2((a+b)^2)^2=[(a-b)(a+b)]^4$; a^2-b^2

$a=\sqrt[4]{\sqrt{6}-2}\xrightarrow{\text{توان ۲}}\sqrt{\sqrt{6}-2}$ $\quad (\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2})^2$

$b=\sqrt[4]{\sqrt{6}+2}\xrightarrow{\text{توان ۲}}\sqrt{\sqrt{6}-2}$ $\quad \sqrt{6}-\cancel{2}+\sqrt{6}+\cancel{2}-2\underbrace{\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)}}_{\sqrt{2}}$

$(2\sqrt{6}-2\sqrt{2})^2=24+8-16\sqrt{3}$

$32-16\sqrt{3}$

$32-16\sqrt{3}=t(2-\sqrt{3})$

$16(2-\sqrt{3})=t(2-\sqrt{3})\to\boxed{t=16}$

7

$a=\sqrt[4]{(2-\sqrt{3})^2}=\sqrt{2-\sqrt{3}}$

$(a+\frac{1}{a}+\sqrt{2})^2(a+\frac{1}{a}-\sqrt{2})^2=[(a+\frac{1}{a}+\sqrt{2})(a+\frac{1}{a}-\sqrt{2})]^2=2^t$

$(a^2+\frac{1}{a^2}+2-2)^2=(a^2+\frac{1}{a^2})^2=2^t$

$a^2=|2-\sqrt{3}|=2-\sqrt{3}$

$\hookrightarrow 2-\cancel{\sqrt{3}}+2+\cancel{\sqrt{3}}=(4)^2=(2^2)^2=2^4=2^t$

$\frac{1}{a^2}=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{2+\sqrt{3}}{1}=2+\sqrt{3}$

$\boxed{t=4}$ ✓

8

$A=\sqrt[15]{2^4\times 2^6}=\sqrt[15]{2^{10}}=\sqrt[3]{2^2}$

$(\frac{1}{2})^{-\frac{4}{3}}=(2)^{\frac{4}{3}}=\sqrt[3]{2^4}$

$\sqrt[3]{2^2\times 2^4}=\sqrt[3]{2^6}=2^2=4=A$

$(2A)^{-\frac{1}{3}}=(8)^{-\frac{1}{3}}=\sqrt[3]{\frac{1}{8}}=\boxed{\frac{1}{2}}$

9

$\sqrt[7]{a}=27\times a^{\frac{15}{7}}$

$a^{\frac{1}{7}}=27\times a^{\frac{15}{7}}\xrightarrow{\div a^{\frac{1}{7}}}1=27\times \cancel{a^{\frac{14}{7}}}a^2\to a^2=\frac{1}{27}\to a=\underset{\text{غ ق ق}}{\pm}\frac{1}{\sqrt{27}}\to a=\frac{1}{\underset{3\sqrt{3}}{\sqrt{27}}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{9}$

$\frac{1}{a}\to\frac{9}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{9\sqrt{3}}{3}=3\sqrt{3}$ $\quad \frac{1}{a}-3=\frac{3\sqrt{3}-3}{1+\sqrt{3}}\times\frac{1-\sqrt{3}}{1-\sqrt{3}}=\frac{3\sqrt{3}-9-3+3\sqrt{3}}{1-3}=\frac{6\sqrt{3}-12}{-2}$

$\boxed{6-3\sqrt{3}}=\frac{2(3\sqrt{3}-6)}{-2}$

10

$(\underbrace{\sqrt{x+a}-\sqrt{x-4}}_{2})(\sqrt{x+a}+\sqrt{x-4})=\cancel{x}+a-\cancel{x}+4$

$\hookrightarrow\frac{4+a}{2}=2+\frac{a}{2}\xrightarrow{-2}\boxed{\frac{a}{2}}$