نمره سوال ۲۰

۱) $(1+9+0+0\cdot\epsilon\cdots)(2+4+0\cdot\epsilon\cdots)$

$7\times 9 = 42$ ✓ ۲

رقم جان: ۲

۲) الف)

$$\sqrt[4]{\frac{1}{9}\times(\epsilon-\sqrt{7})(\epsilon+\sqrt{7})\times 2} = \sqrt[4]{2}$$ ۴

ب) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}(\sqrt{5}+\sqrt{2})} = \frac{1}{\sqrt{2}}$ ✓ ۲

$t^2 = 2-\sqrt{5}+3+\sqrt{5}-4 = 1$

$|t| = \sqrt{2}$

$-\sqrt{2}\times\frac{1}{\sqrt{2}} = -1$ ۳

۳) الف) $\frac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}} \propto \frac{5+\sqrt{6}}{5+\sqrt{6}}$

$-2\frac{1}{\sqrt{2}-1} \propto \frac{\sqrt{2}+1}{\sqrt{2}+1}$ ✓ ۲

$\longrightarrow \frac{19\sqrt{2}+19\sqrt{3}}{19} - \sqrt{3} - 1 = \sqrt{2}-1$

ب) $\frac{\sqrt{27}-1}{\epsilon+\sqrt{3}} \propto \frac{\epsilon-\sqrt{3}}{\epsilon-\sqrt{3}} = \frac{13\sqrt{3}-13}{13} = \sqrt{3}-1$

$(2-\sqrt{3})^{-1} = \frac{1}{(2-\sqrt{3})} \propto \frac{(2+\sqrt{3})}{(2+\sqrt{3})} = 2+\sqrt{3}$

$\xrightarrow{+} 2\sqrt{3}+1$

۴) الف) $\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{13}-1}}{\sqrt{\sqrt{2}-\sqrt{2}}} = t$

$t^2 = \frac{1+\sqrt{3}-1+\sqrt{3}+2\sqrt{2}(\sqrt{2}-\sqrt{2})}{\sqrt{2}-\sqrt{2}\times\sqrt{2}+\sqrt{2}} \xrightarrow{\text{ضرب}} \sqrt{6}+2$

$t-2 = \sqrt{6}+2-4 = \sqrt{6}$ ✓ ۲

۱۲ $= 3 \times 2^{\frac{24}{12}} = 3 \times 2^{\frac{1}{12}} \times 2^{\frac{1}{4}} \times 2 \times 3^{\frac{2}{3}}$ (۱

(۳)

$$\frac{2^n(1+2^3+9+2^4+2^5+2^6)}{2^n(\frac{1}{4}+\frac{1}{2}+1+2+8+16)} = 25$$

$$\frac{3^n \times 3^4 \times 4}{3^n \times \frac{3^4}{4}} = 25 \xrightarrow{\div 25} \frac{3^n \times 27}{3^n \times \frac{3^4}{4}} = 1$$ ✓ (۲)

$$\frac{3^n \times 2^8}{2^n \times 2^6} = 1 \rightsquigarrow (\frac{3}{2})^n = \frac{9}{4} \Rightarrow n=2$$

(۴)

$$\left[(a^4+b^4+2a^2b^2)-4a^2b^2\right]^2 =$$

$$\sqrt{6}-2+\sqrt{6}+2-2(\sqrt{\sqrt{6}+2})(\sqrt{\sqrt{6}-2}) =$$ ✓ (۲)

$$(2\sqrt{6}-2\sqrt{2})^2 = 4(18-2\sqrt{12}) \Rightarrow 16(2-\sqrt{3})$$

$$t=16$$

(۷)

$$\left[(a^2+\frac{1}{a^2}+2-2)\right]^2 = \left[\frac{1}{a^2}+a^2\right]^2 =$$

$$\frac{1}{a^4}+a^4+2 = 7-4\sqrt{3}+\frac{1}{7+4\sqrt{3}}+2$$

$$= 7-4\sqrt{3}+\frac{7+4\sqrt{3}}{49-48}+2 \Rightarrow 16 = 2^t$$

$$t=4$$ ✓ (۲)

(١) $2^{\frac{3}{2}} \times 2^{\frac{3}{3}} \times 2^{\frac{3}{2}} = 2^{\frac{9}{3}} = 4$

$(2 \times 4)^{-\frac{1}{2}} = (\frac{1}{8})^{\frac{1}{2}} = 2^{-1}$ ✓ (٢)

(٩) $\sqrt{3}\sqrt{3} = 3$ $\quad a^{-2} = 3^{3}$ $\quad \frac{1}{a^2} = 3$

$\frac{3(\sqrt{2}-1)}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{9+3-6\sqrt{2}}{2} = 6-3\sqrt{2}$ ✓ (٢)

(١٥)

$\sqrt{n+a} - \sqrt{n-4} = 2$ $\xrightarrow[\text{مرافق}]{\text{ضرب في}}$

$n+a-n+4 = 2(\sqrt{n+a} + \sqrt{n-4})$

$\frac{a}{2} + 2 = \sqrt{n+a} + \sqrt{n-4}$

$\frac{a}{2} = \sqrt{n+a} + \sqrt{n-4} - 2$ ✓ (٢)