تمرین سری ۱

①

$(1+9+0+0+\cdots)(2+4+0+\cdots)$

$7 \times 6 = 42$

رقم یکان: ۲

②

الف)

$$\sqrt[4]{\frac{1}{9} \times (4-\sqrt{7})(4+\sqrt{7}) \times 2} = \sqrt[4]{2}$$

ب) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}(\sqrt{3}+\sqrt{2})} = \frac{1}{\sqrt{2}}$

$t^2 = 3-\sqrt{3}+3+\sqrt{3}-4 = 2$

$|t| = \sqrt{2}$

$-\sqrt{2} \times \frac{1}{\sqrt{2}} = -1$

③

الف) $\frac{2\sqrt{2}+3\sqrt{3}}{3-\sqrt{6}} \propto \frac{3+\sqrt{6}}{3+\sqrt{6}}$

$-2\frac{1}{\sqrt{2}-1} \propto \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$\longrightarrow \frac{19\sqrt{2}+19\sqrt{3}}{19} - \sqrt{3} - 1 = \sqrt{2}-1$

ب) $\frac{\sqrt{27}-1}{4+\sqrt{3}} \propto \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{13\sqrt{3}-13}{13} = \sqrt{3}-1$

$(2-\sqrt{3})^{-1} = \frac{1}{(2-\sqrt{3})} \propto \frac{(2+\sqrt{3})}{(2+\sqrt{3})} = 2+\sqrt{3}$

$\xrightarrow{+} 2\sqrt{3}+1$

④

الف) $\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{13}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} = t$

$t^2 = \frac{1+\sqrt{3}-1+\sqrt{3}+2\sqrt{2}(\sqrt{3}-\sqrt{2})}{\sqrt{3}-\sqrt{2} \times \sqrt{3}+\sqrt{2}} \xrightarrow{\text{ضرب}} \sqrt{6}+2$

$t-2 = \sqrt{6}+2-2 = \sqrt{6}$

۱) $۲^{\frac{۲}{۳}} \times ۳ \times ۲^{\frac{۱}{۱۲}} \times ۲ \times ۲^{\frac{۱}{۴}} \times ۳ = ۲^{\frac{۲۴}{۱۲}} \times ۳ = ۱۲$

③

$$\frac{۲^{n}(۱+۳+۹+۲^{۳}+۲^{۴}+۲^{۵})}{۲^{n}(\frac{۱}{۴}+\frac{۱}{۲}+۱+۲+۴+۸)} = ۵۲$$

$$\frac{۳^{n} \times ۳۶۴}{۲^{n} \times \frac{۶۳}{۴}} = ۵۲ \xrightarrow{\div ۵۲} \frac{۳^{n} \times ۷}{۲^{n} \times \frac{۶۳}{۴}} = ۱$$

$$\frac{۳^{n} \times ۲۸}{۲^{n} \times ۶۳} = ۱ \rightsquigarrow (\frac{۳}{۲})^{n} = \frac{۹}{۴} \Rightarrow \boxed{n=۲}$$

④

$$\left[(a^{۴}+b^{۴}+۲a^{۲}b^{۲}) - ۴a^{۲}b^{۲}\right]^{۲} =$$

$$\sqrt{۶}-۲+\sqrt{۶}+۲-۲(\sqrt{\sqrt{۶}+۲})(\sqrt{\sqrt{۶}-۲}) =$$

$$(۲\sqrt{۶}-۲\sqrt{۲})^{۲} = ۴(۱۸-۲\sqrt{۱۲}) \Rightarrow ۱۶(۲-\sqrt{۳})$$

$$\boxed{t=۱۶}$$

⑦

$$\left[(a^{۲}+\frac{۱}{a^{۲}}+۲-۲)\right]^{۲} = \left[\frac{۱}{a^{۲}}+a^{۲}\right]^{۲} =$$

$$\frac{۱}{a^{۴}}+a^{۴}+۲ = ۷-۴\sqrt{۳}+\frac{۱}{۷+۴\sqrt{۳}}+۲$$

$$= ۷-۴\sqrt{۳}+\frac{۷+۴\sqrt{۳}}{۴۹-۴۸}+۲ \Rightarrow ۱۶ = ۲^{t}$$

$$\boxed{t=۴}$$

$2^{\frac{2}{3}} \times 2^{\frac{4}{3}} \times 2^{\frac{3}{2}} = 2^{\frac{13}{6}} = 4$ ①

$(2 \times 4)^{-\frac{1}{2}} = \left(\frac{1}{8}\right)^{\frac{1}{2}} = \boxed{2^{-1}}$

⑨ $\sqrt{3}\sqrt[3]{3} = 3^{\frac{1}{2}} \times 3^{\frac{1}{3}} = 3^{\frac{5}{6}}$ $\quad \frac{1}{a} = a^{-2} = 3^{\frac{5}{6}}$

$\frac{3(\sqrt{2}-1)}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{6\sqrt{2}-6+3}{2} = \boxed{6-3\sqrt{2}}$

⑩

$\sqrt{n+a} - \sqrt{n-4} = 2$ $\xrightarrow[\text{مرفع}]{\text{ضرب}}$

$n+a-n+4 = 2(\sqrt{n+a} + \sqrt{n-4})$

$\frac{a}{2} + 2 = \sqrt{n+a} + \sqrt{n-4}$

$\boxed{\frac{a}{2} = \sqrt{n+a} + \sqrt{n-4} - 2}$