پرشاری ۲۰ کلاس دهم دختر تکلیف ۲۱

۱- $(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+26!)=$

رقم یکان؟ مثلا: $1! \to 1$, $2! \to 2$, $3! \to 6$, $4! \to 24$, $5! \to 0$

از اینجا به بعد بخاطر اینکه هم ۲ هم ۵ داریم رقم یکان صفر است.

۲ ✓

$$(\underbrace{1+6+0+\dots+0}_{7})(\underbrace{2+4+0+\dots+0}_{6}) = 7\times 6 = 42$$

جواب آخر

۲- الف) $\sqrt[4]{(4+\sqrt7)^{-1}}$

$$\frac{\sqrt{1+\sqrt7}}{\sqrt[4]{(1+\sqrt7)^2}} = \sqrt[4]{8+2\sqrt7}$$

$$= \sqrt[4]{\frac{8+2\sqrt7}{4+\sqrt7}} = \boxed{\sqrt[4]{2}}$$

۲ ✓

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\underbrace{\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)}_{A}$

$$\frac{\sqrt2+\sqrt5}{\sqrt2(\sqrt5+\sqrt2)} = \frac{1}{\sqrt2} = \frac{\sqrt2}{2}$$

$$A^2 = 3-\sqrt5+3+\sqrt5-2\sqrt{(3-\sqrt5)\times(3+\sqrt5)}$$

$$= 6-2\sqrt{\underbrace{9-5}_{4}} = 6-4 = 2 \Rightarrow A = \pm\sqrt2$$

چون رادیکال دوم از رادیکال اول بزرگتر است.

$$\frac{\sqrt2}{2}\times -\sqrt2 = \boxed{-1}$$

۳- الف) $\frac{\sqrt8+\sqrt{27}}{5-\sqrt6} - 2\left(\sqrt[4]{9}-1\right)^{-1}$

$$\frac{2\sqrt2+3\sqrt3}{5-\sqrt6}\times\frac{(5+\sqrt6)}{(5+\sqrt6)} = \frac{10\sqrt2+4\sqrt3+15\sqrt3+9\sqrt2}{19}$$

$$= \frac{19(\sqrt2+\sqrt3)}{19} = \sqrt2+\sqrt3 - 2\left(\frac{1}{\sqrt3-1}\right) = \sqrt2+\sqrt3-\sqrt3-1$$

$$\frac{1}{\sqrt3-1}\times\frac{\sqrt3+1}{\sqrt3+1} = \frac{\sqrt3+1}{2}$$

$$= \boxed{\sqrt2-1}$$

جواب آخر

۳ ۲ ✓

ب) $\frac{\sqrt{27}-1}{4+\sqrt3} + (2-\sqrt3)^{-1}$

$$\frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3} = \frac{2+\sqrt3}{4-3} = 2+\sqrt3$$

$$\frac{3\sqrt3-1}{4+\sqrt3}\times\frac{(4-\sqrt3)}{(4-\sqrt3)} = \frac{12\sqrt3-9-4+\sqrt3}{13} = \frac{13\sqrt3-13}{13} = \frac{13(\sqrt3-1)}{13} = \sqrt3-1$$

$$\Rightarrow \sqrt3-1+2+\sqrt3 = \boxed{2\sqrt3+1}$$

۴- الف) $\frac{\left(\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}\right)}{\sqrt{\sqrt3-\sqrt2}}-2$ ، $\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}=A$

جواب جمع دو رادیکال بزرگتر از صفر است ✓

$$A^2 = 1+\sqrt3+\sqrt3-1+2\sqrt{\underbrace{(\sqrt3+1)(\sqrt3-1)}_{3-1=2}} = 2\sqrt3+2\sqrt2 \Rightarrow A = \pm\sqrt{2\sqrt3+2\sqrt2}$$

$$\sqrt{\frac{2(\sqrt3+\sqrt2)}{\sqrt3-\sqrt2}\times\frac{(\sqrt3+\sqrt2)}{(\sqrt3+\sqrt2)}} = \sqrt{\frac{2(\sqrt3+\sqrt2)^2}{1}} = (\sqrt3+\sqrt2)\sqrt2 = \sqrt6+2 \Rightarrow \sqrt6+2-2$$

$$= \boxed{\sqrt6}$$

جواب آخر

۲ ✓

۴ ب) $\sqrt[4]{\sqrt[3]{2^8}} \times \sqrt[4]{162} \times \sqrt{4\sqrt[6]{2}} = 2^{\frac{2}{3}} \times 3 \times 2^{\frac{1}{4}} \times 2 \times 2^{\frac{1}{12}}$

$\sqrt[3]{2^8}$ … $= \sqrt[3]{2^2} = \sqrt[3]{4} \Rightarrow 2^{\frac{2}{3}}$

$\sqrt[4]{3^4 \times 2} = 3\sqrt[4]{2} \Rightarrow 3 \times 2^{\frac{1}{4}}$

$\sqrt{\sqrt[6]{2^{13}}} = \sqrt[12]{2^{13}} = 2\sqrt[12]{2} \Rightarrow 2 \times 2^{\frac{1}{12}}$

$= 3 \times 2 \times 2 = \boxed{12}$

۵-

$$\frac{3^x(1+3+9+27+81+243)}{2^x(\frac{1}{4}+\frac{1}{2}+1+2+4+8)} = 52 \Rightarrow \left(\frac{3}{2}\right)^x \times \frac{364}{15.75} = 52$$

$$\Rightarrow \left(\frac{3}{2}\right)^x = \frac{52 \times 15.75}{364} = 2.25$$

$(1.5)^x = 2.25 \Rightarrow \boxed{x=2}$

(۲)

۶-

$$\left((a-b)^2(a+b)^2\right)^2 = (a^2-b^2)^4 = \left(\sqrt{\sqrt{6}-2} - \sqrt{\sqrt{6}+2}\right)^4 = \left(\left(\sqrt{\sqrt{6}-2} - \sqrt{\sqrt{6}+2}\right)^2\right)^2$$

ضرب

$$= \left(2\sqrt{6} - 2\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)}\right)^2 = (2\sqrt{6}-2\sqrt{2})^2$$

$$4(\sqrt{6}-\sqrt{2})^2 = 4(8-2\sqrt{12})$$

$8-4\sqrt{3}$

$$= 32-16\sqrt{3} \Rightarrow 16(2-\sqrt{3})$$

$\Rightarrow \boxed{t=16}$

(۲)

۷-

ضرب

$$\left(\left(a+\frac{1}{a}\right)^2 - (\sqrt{2})^2\right)^2 = \left(a^2+\frac{1}{a^2}+2-2\right)^2 = \left(a^2+\frac{1}{a^2}\right)^2$$

$$= a^4+\frac{1}{a^4}+2 = 7-4\sqrt{3} + \frac{1}{7-4\sqrt{3}} + 2 = 14+2 = 16$$

$\frac{1}{7-4\sqrt{3}} \to \frac{7+4\sqrt{3}}{1}$ ، $49-48$

$a = \sqrt[4]{7-4\sqrt{3}} \Rightarrow a^4 = 7-4\sqrt{3}$ ، $\Rightarrow 16 = 2^t \Rightarrow \boxed{t=4}$

(۲)

۸-

$A = \sqrt[5]{4\sqrt[3]{16}}\left(\frac{1}{2}\right)^{-\frac{4}{3}}$, $(2A)^{-\frac{1}{3}} = ?$

$2^{\frac{2}{5}} \times 2^{\frac{4}{15}} \times 2^{\frac{4}{3}} = A = 2^{\frac{30}{15}} = 2^2 = 4$

$(8)^{-\frac{1}{3}} = \left(\frac{1}{8}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{1}{8}} = \boxed{\frac{1}{2}}$

(۲)

۹-

$$\sqrt[7]{a} = 27a^{\frac{15}{7}} \to a^{\frac{1}{7}} = 27 \times a^{\frac{15}{7}} \Rightarrow \frac{a^{\frac{1}{7}}}{a^{\frac{15}{7}}} = 27 \Rightarrow a^{-2} = 27 = 3^3$$

$\Rightarrow a = 3^{-\frac{3}{2}}$

$$\frac{1}{a} = \frac{1}{3^{-\frac{3}{2}}} = 3^{\frac{3}{2}} = 3\sqrt{3}$$

(۲)

$$\frac{3\sqrt{3}-3}{1+\sqrt{3}} \times \frac{(1-\sqrt{3})}{(1-\sqrt{3})} = \frac{3(\sqrt{3}-1)\overbrace{(1-\sqrt{3})}^{-(\sqrt{3}-1)}}{-2} = \frac{-3}{-2} \times (\sqrt{3}-1)^2 = \frac{3}{2}(4-2\sqrt{3})$$

$= \boxed{6-3\sqrt{3}}$

۱۰-

$\sqrt{x+a} - \sqrt{x-4} = 2$, $\underbrace{\sqrt{x+a} + \sqrt{x-4}}_{A} - 2 = ? \Rightarrow \frac{a}{2}+2-2 = \boxed{\frac{a}{2}}$

$$(\sqrt{x+a} - \sqrt{x-4})(\sqrt{x+a} + \sqrt{x-4}) = 2A = a+4 \Rightarrow A = \frac{a}{2}+2$$

(۲)