۱) $1! = 1 + 3! = 6 + 5! = 120 = 7 \quad 2! = 2 + 4! = 24 + 6! = 720 = 6$

$7 \times 6 = 42 \longrightarrow$ رقم یکان: $\boxed{2}$

۲ ✓

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۲) الف) $\left(\sqrt{1+\sqrt7}\right)^1 \longrightarrow \sqrt[4]{(1+\sqrt7)^2} \Rightarrow \sqrt[4]{8+2\sqrt7} \times \sqrt[4]{\frac{1}{4+\sqrt7}} = \sqrt[4]{2}$

ب) $\frac{(\sqrt2+\sqrt5)}{\sqrt2(\sqrt5+\sqrt2)} \times \left(\sqrt{3-\sqrt5} - \sqrt{3+\sqrt5}\right) = \sqrt{\frac{6-2\sqrt5}{2}} - \sqrt{\frac{6+2\sqrt5}{2}}$

$\frac{\sqrt2}{2}$ ، $(\sqrt5-1)^2$ ، $(\sqrt5+1)^2$

$\boxed{-1} = \frac{\sqrt5-1-\cancel{\sqrt5}-1}{2}$

۲ ✓

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۳) الف) $\frac{2\sqrt2+3\sqrt3}{\sqrt5-\sqrt6} \times \frac{\sqrt5+\sqrt6}{\sqrt5+\sqrt6} = \frac{\cancel{12}(\sqrt2+\sqrt3)}{\cancel{5-6}} = \sqrt2+\sqrt3$

$-2\left(\frac{1}{\sqrt3-1}\right) \times \frac{\sqrt3+1}{\sqrt3+1} = \frac{-2(\sqrt3+1)}{2} = -(\sqrt3+1)$

$\rightarrow \sqrt2+\sqrt3-\sqrt3-1 = \boxed{\sqrt2-1}$

ب) $\frac{3\sqrt3-1}{4+\sqrt3} \times \frac{4-\sqrt3}{4-\sqrt3} = \sqrt3-1 \quad \frac{1}{2-\sqrt3} \times \frac{2+\sqrt3}{2+\sqrt3} = \frac{2+\sqrt3}{4-3} = 2+\sqrt3$

$\sqrt3 + \sqrt3 + 2 - 1 = \boxed{2\sqrt3-1}$

۲ ✓

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۴) الف) $\left(\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}\right)^2 - 2 = \frac{1+\sqrt3+\sqrt3-1+2\sqrt2}{\sqrt3-\sqrt2} = \frac{2\sqrt3+2\sqrt2}{\sqrt3-\sqrt2}$

$\frac{2(\sqrt3+\sqrt2)}{\sqrt3-\sqrt2} \times \frac{(\sqrt3+\sqrt2)}{(\sqrt3+\sqrt2)} = \frac{2(\sqrt3+\sqrt2)^2}{3-2} = 2(\sqrt3+\sqrt2)^2 = 4\sqrt6+10$

$\sqrt{4\sqrt6+10} = \sqrt{(2+\sqrt6)^2} = 2+\sqrt6-2 = \boxed{\sqrt6}$

۲ ✓

ب) $2^{\frac{8}{12}} \times 2^{\frac{4}{12}} \times 2^{\frac{12}{12}} \times 3^{\frac{12}{12}} = 2^{\frac{24}{12}} \times 3^{\frac{12}{12}} = 2^2 \times 3 = 4 \times 3 = \boxed{12}$

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۵) $\frac{3^x(1+3+9+27+81+243)}{2^{x-2}(1+2+4+8+16+32)} = \frac{3^x \times \cancel{364}}{2^{x-2} \times 639} = 52 \Rightarrow 3^{x-2} = 2^{x-2}$

$\boxed{x=2}$

۲ ✓

۶

$a^۲+b^۲=t \longrightarrow (t-۲ab)^۲(t+۲ab)^۲=(t^۲-۴a^۲b^۲)^۲ \longrightarrow$

$(a^۴+b^۴+\underbrace{۲a^۲b^۲-۴a^۲b^۲}_{-۲a^۲b^۲})^۲ \longrightarrow (a^۲-b^۲)^۲ \longrightarrow (\sqrt{\sqrt{۵}-۲}-\sqrt{\sqrt{۵}+۲})^۲$

پایین صفحه: ۱

۷

$((a+\frac{۱}{a})^۲-۲)^۲=۲^t \qquad a=\sqrt[۴]{۷+۴\sqrt{۳}} \Rightarrow a^۴=۷+۴\sqrt{۳} \Rightarrow \frac{۱}{a^۴}=\frac{۱}{۷+۴\sqrt{۳}}$

$\hookrightarrow (a^۲+\frac{۱}{a^۲}+\cancel{۲}-\cancel{۲})^۲=a^۴+\frac{۱}{a^۴}+۲=۲^t \longrightarrow ۷+۴\sqrt{۳}+\frac{۱}{۷+۴\sqrt{۳}}+۲=۲^t$

۳ ✓

$\frac{۴۹+۴۸-۵۶\sqrt{۳}+۱+۱۴-۸\sqrt{۳}}{۷-۴\sqrt{۳}}=\frac{۱۱۲-۶۴\sqrt{۳}}{۷-۴\sqrt{۳}}=\frac{۱۶(۷-\cancel{۴\sqrt{۳}})}{(۷-\cancel{۴\sqrt{۳}})}=۲^t \Rightarrow \boxed{t=۴}$

۸

$A=۲^{\frac{۲}{۳}}\times ۲^{\frac{۲}{۳}}=۲^{\frac{۴}{۳}}=۴ \Rightarrow (۸)^{-\frac{۱}{۳}} \to (۲^۳)^{-\frac{۱}{۳}}=\boxed{\frac{۱}{۲}}$

۳ ✓

۹

$a^{\frac{۱}{۷}}=۲۷\times a^{\frac{۱۵}{۷}} \longrightarrow ۱=۲۷\times a^{\frac{۱۴}{۷}} \longrightarrow ۱=۲۷\times a^۲ \to a=\frac{۱}{\sqrt{۲۷}}$

۳ ✓

$\frac{(\frac{۱}{a}-۳)}{\sqrt{۳}+۱} \to \frac{۳\sqrt{۳}-۳}{\sqrt{۳}+۱}\times\frac{\sqrt{۳}-۱}{\sqrt{۳}-۱} \to \frac{۳(\sqrt{۳}-۱)^۲}{۲}=\frac{۳\times۲(۲-\sqrt{۳})}{\cancel{۲}}=\boxed{۶-۳\sqrt{۳}}$

۱۰

$(\sqrt{x+a}-\sqrt{x-۴})(\sqrt{x+a}+\sqrt{x-۴})=(\cancel{x}+a-\cancel{x}+۴)=۲(\sqrt{x+a}+\sqrt{x+۴})$

$a+۴=۲(\sqrt{x+a}+\sqrt{x+۴}) \longrightarrow \frac{a}{۲}+۲=\sqrt{x+a}+\sqrt{x+۴}$

$\sqrt{x+a}+\sqrt{x+۴}-۲=\frac{a}{۲}+۲-۲=\boxed{\frac{a}{۲}}$

۳ ✓

۴_ ادامه:

$$\left(\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2}\right)^4$$

$$\left(\sqrt{6}-2+\sqrt{6}+2-2\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)}\right)^2=\left(2\sqrt{6}-2\sqrt{2}\right)^2$$

$$4\left(6+2-2\sqrt{12}\right)=16\left(2-\sqrt{3}\right)$$

$$\boxed{t=16}$$