

۱۹, ۷۵

$$(1! + 2! + 3! + \dots + 24!) (1! + 2! + 3! + \dots + 24!)$$

از ۱ تا ۲۴ شامل ۲۴ عدد در صفر می شود

از ۱ تا ۲۴ شامل ۲۴ عدد دارد
۲۴ رقم

$$1! + 2! + 3! + \dots + 24! = 7$$

$$2! + 3! + \dots + 24! = 24$$

$$\rightarrow 7 \times 24 = 168$$

✓

$$\sqrt{1 + \sqrt{5}} - 1 \times \sqrt{1 + \sqrt{5}} \rightarrow \sqrt{\frac{1}{1 + \sqrt{5}}} \times (1 + \sqrt{5})^2 \rightarrow \sqrt{\frac{1 + 2\sqrt{5} + 5}{1 + \sqrt{5}}} = \sqrt{2}$$

$$\text{ب) } \left(\frac{\sqrt{1 + \sqrt{5}}}{\sqrt{1 + \sqrt{5}}} \right) (\sqrt{3 - \sqrt{5}} - \sqrt{3 + \sqrt{5}}) \cdot \frac{\sqrt{2}}{2} (\sqrt{3 - \sqrt{5}} - \sqrt{3 + \sqrt{5}}) \rightarrow (\sqrt{3 - \sqrt{5}} - \sqrt{3 + \sqrt{5}})^2$$

$$= 3 - \sqrt{5} + 3 + \sqrt{5} - 2(\sqrt{3 - \sqrt{5}} \cdot \sqrt{3 + \sqrt{5}}) = 2 \rightarrow \frac{\sqrt{2}}{2} (\sqrt{3 - \sqrt{5}} - \sqrt{3 + \sqrt{5}}) = \frac{\sqrt{2}}{2} \times (-\sqrt{2})$$

۲ - ۱ = ۱

$$= -1$$

✓

$$\text{الف) } \frac{3\sqrt{3} + 2\sqrt{2}}{5 - \sqrt{2}} \times \frac{5 + \sqrt{2}}{5 + \sqrt{2}} = \frac{15\sqrt{3} + 10\sqrt{6} + 10\sqrt{3} + 4\sqrt{2}}{25 - 2} = \frac{25\sqrt{3} + 10\sqrt{6} + 4\sqrt{2}}{23}$$

$$\rightarrow 2(\sqrt{3} - 1)^{-1} = 2(\sqrt{3} - 1)^{-1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{2}{\sqrt{3} + 1} \cdot \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{2(\sqrt{3} + 1)}{3 - 1} = \sqrt{3} + 1$$

$$\text{ب) } \frac{3\sqrt{3} - 1}{5 - \sqrt{2}} \times \frac{5 + \sqrt{2}}{5 + \sqrt{2}} = \frac{15\sqrt{3} - 5 + 5\sqrt{6} - \sqrt{2}}{25 - 2} = \frac{15\sqrt{3} - 5 + 5\sqrt{6} - \sqrt{2}}{23}$$

$$\rightarrow (2 - \sqrt{3})^{-1} = \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3}$$

در عبارت سوال

$$\sqrt{1 + \sqrt{3}} + \sqrt{3 - 1}$$

۲ - ۱

صورت سوال ۲ - داشت

$$\text{الف) } \left(\frac{\sqrt{3} + 1}{\sqrt{3} - 1} + \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \right) \times \frac{\sqrt{2}(\sqrt{3} + \sqrt{2})}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} = 2(\sqrt{3} + \sqrt{2}) \times \frac{\sqrt{2}(\sqrt{3} + \sqrt{2})}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} = 2(\sqrt{3} + \sqrt{2})^2 = 2(3 + 2\sqrt{6} + 2) = 2(5 + 2\sqrt{6}) = 10 + 4\sqrt{6}$$

$$\text{ب) } \sqrt{2^8} \times \sqrt{3^4} \times \sqrt{2^4} \times \sqrt{2^4} = \sqrt{2^8 \times 3^4 \times 2^4 \times 2^4} = \sqrt{2^{16} \times 3^4} = 2^8 \times 3^2 = 256 \times 9 = 2304$$

۱, ۷۵

$$\frac{3^n (1 + 3 + 9 + 27 + \dots + 81 + 243)}{2^{n-2} (1 + 2 + 4 + 8 + 16 + 32)} = 27 \rightarrow \frac{3^n \times 27}{2^{n-2} \times 63} = 27 \rightarrow \frac{3^n}{2^{n-2}} \times \frac{1}{9} = 1$$

$$\rightarrow \left(\frac{3}{2} \right)^n = \frac{9}{2} \rightarrow n = 2$$

✓

$$(a^2 + b^2 - rab)^2 (a^2 + b^2 + rab)^2 = ((a-b)^2)^2 ((a+b)^2)^2 = ((a-b)(a+b))^2 = (a^2 - b^2)^2$$

$$= (a^4 - 2a^2b^2 + b^4)^2 = (\sqrt{4} - 2 + \sqrt{(\sqrt{4}-1)(\sqrt{4}+1)} + \sqrt{4} + 1)^2 = (2\sqrt{4} - 2\sqrt{4})^2$$

$$= (\sqrt{4} - \sqrt{4})^2 = 0 \quad (4 + 2 - 2\sqrt{4}) = 4(2 - \sqrt{4}) \Rightarrow \boxed{6.14}$$

✓

$$(a + \frac{1}{a} + \sqrt{r})^2 (a + \frac{1}{a} - \sqrt{r})^2 = ((a + \frac{1}{a})^2 - (\sqrt{r})^2)^2 = (a^2 + \frac{1}{a^2} + 2 - r)^2 = a^4 + \frac{1}{a^4} + 2 + r - 2\sqrt{r}$$

$$a^4 + \frac{1}{a^4} + 2 + r - 2\sqrt{r} + \frac{1}{\sqrt{4} - \sqrt{r}} + r = (1 + \sqrt{4} - \sqrt{r}) + (\frac{1}{\sqrt{4} - \sqrt{r}} + \frac{\sqrt{4} - \sqrt{r}}{\sqrt{4} + \sqrt{r}}) + r$$

$$\Rightarrow 1 + \sqrt{4} - \sqrt{r} + 1 + \sqrt{4} + r = 4 + 2 - 2\sqrt{r} \Rightarrow \boxed{6.14}$$

✓

$$A = \sqrt[3]{\sqrt[3]{14}} = (\frac{1}{r})^{-\frac{1}{r}} = \sqrt[3]{\frac{1}{r}} = r^{\frac{1}{3}} = r^{\frac{1}{3}} \times r^{\frac{1}{3}} = r^{\frac{2}{3}}$$

$$\rightarrow (rA)^{-\frac{1}{r}} = \sqrt[3]{\frac{1}{rA}} = \sqrt[3]{\frac{1}{r}} = \frac{1}{r}$$

✓

$$\sqrt[3]{a} = r \sqrt[3]{a} \frac{10}{r} \xrightarrow{\sqrt[3]{10}} a = r \sqrt[3]{a} \times a^{10} \rightarrow a^{11} = (r^3)^{10} \rightarrow a = r^{-\frac{10}{r}} \rightarrow a = \frac{1}{\sqrt[3]{r}}$$

$$\frac{1}{a} - r = r\sqrt{r} - r = r(\sqrt{r} - 1)$$

$$\frac{\frac{1}{a} - r}{\sqrt{r} + 1} = \frac{\sqrt{r} - 1}{\sqrt{r} - 1} = \frac{r(\sqrt{r} - 1)(\sqrt{r} - 1)}{r - 1} = \frac{r(2 - 2\sqrt{r})}{r - 1} = r(2 - 2\sqrt{r})$$

$$= \boxed{4 - 2\sqrt{r}}$$

✓

$$(\sqrt{na} - \sqrt{n-2})(\sqrt{na} + \sqrt{n-2}) = na - n + 2 = a + 2 = r(\sqrt{na} + \sqrt{n-2})$$

$$\sqrt{na} + \sqrt{n-2} = \frac{a+2}{r} \rightarrow \sqrt{na} + \sqrt{n-2} = r, \frac{a+2}{r} = r, \frac{a}{r}$$

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