/ /

۱- $(۱+۶+۱۰+\dots+۵)-(۲+۴+۶+\dots+۵)=$ ۴

۷×۶

۱۹,۷۵

۲- الف) $\sqrt[۴]{\frac{۱}{۴+\sqrt{۷}}} \times \sqrt[۴]{(۱+\sqrt{۷})^۲} = \sqrt[۴]{\frac{۱+۷+۲\sqrt{۷}}{۴+\sqrt{۷}}} = \sqrt[۴]{\frac{۸+۲\sqrt{۷}}{۴+\sqrt{۷}}} = \sqrt[۴]{\frac{۲(۴+\sqrt{۷})}{۴+\sqrt{۷}}} = \sqrt[۴]{۲} = \frac{\sqrt{۲}}{۲}$

ب) $\frac{۲+\sqrt{۵}}{\sqrt{۱۰}+۲} \times \frac{\sqrt{۱۰}-۲}{\sqrt{۱۰}-۲} = \frac{۲\sqrt{۵}+۵\sqrt{۲}-۲\sqrt{۲}-۲\sqrt{۵}}{۱۰-۴} = \frac{۳\sqrt{۲}}{۶} = \frac{\sqrt{۲}}{۲}$

$\frac{\sqrt{۲}}{۲}\left(\sqrt{۳-\sqrt{۵}}-\sqrt{۳+\sqrt{۵}}\right) = \frac{\sqrt{۶-۲\sqrt{۵}}-\sqrt{۶+۲\sqrt{۵}}}{۲} = \frac{\sqrt{(\sqrt{۵}-۱)^۲}-\sqrt{(\sqrt{۵}+۱)^۲}}{۲} = \frac{|\sqrt{۵}-۱|-|\sqrt{۵}+۱|}{۲}$

$= \frac{\sqrt{۵}-۱-\sqrt{۵}-۱}{۲} = \frac{-۲}{۲} = -۱$

۳- الف) $\frac{۲\sqrt{۲}+۳\sqrt{۳}}{۵-\sqrt{۶}} \times \frac{۵+\sqrt{۶}}{۵+\sqrt{۶}} = \frac{۱۰\sqrt{۲}+۲\sqrt{۲}+۱۵\sqrt{۳}+۳\sqrt{۱۸}}{۱۹} = \frac{۱۰\sqrt{۲}+۴\sqrt{۲}+۱۵\sqrt{۳}+۹\sqrt{۲}}{۱۹}$

$= \frac{۱۹\sqrt{۲}+۱۹\sqrt{۳}}{۱۹} = \sqrt{۲}+\sqrt{۳} - \frac{۲}{\sqrt{۳}-۱} = \frac{\sqrt{۶}-\sqrt{۲}+۳-\sqrt{۳}-۲}{\sqrt{۳}-۱} = \frac{\sqrt{۶}-\sqrt{۲}-\sqrt{۳}+۱}{\sqrt{۳}-۱}$

$= \frac{(\sqrt{۳}-۱)(\sqrt{۲})-(\sqrt{۳}-۱)}{\sqrt{۳}-۱} = \sqrt{۲}-۱$

ب) $\frac{۳\sqrt{۲}-۱}{۴+\sqrt{۳}} \times \frac{۴-\sqrt{۳}}{۴-\sqrt{۳}} = \frac{۱۲\sqrt{۲}-۹-۴+\sqrt{۳}}{۱۶-۳} = \sqrt{۳}-۱$

$\frac{۱}{۲-\sqrt{۳}} \times \frac{۲+\sqrt{۳}}{۲+\sqrt{۳}} = ۲+\sqrt{۳}$ $\qquad ۲+\sqrt{۳}+\sqrt{۳}-۱ = ۲\sqrt{۳}+۱$

۴- الف) $\frac{\sqrt{۱+\sqrt{۳}}+\sqrt{\sqrt{۳}-۱}}{\sqrt{\sqrt{۳}-\sqrt{۲}}} - ۲ = \sqrt{۶}+۲-۲ = \sqrt{۶}$

$A^۲ = \frac{۱+\sqrt{۳}+\sqrt{۳}-۱+۲\sqrt{\sqrt{۳}-۱}}{\sqrt{۳}-\sqrt{۲}} = \frac{۲\sqrt{۳}+۲\sqrt{۲}}{\sqrt{۳}-\sqrt{۲}} \times \frac{\sqrt{۳}+\sqrt{۲}}{\sqrt{۳}+\sqrt{۲}} = ۶+۲\sqrt{۲}+۲\sqrt{۶}+۴ = ۱۰+۴\sqrt{۲}$

$= (\sqrt{۶}+۲)^۲ \longrightarrow A = \sqrt{۶}+۲$

ب) $\sqrt[۱۲]{۲^۸} \times \sqrt[۶]{۱۶۲} \times \sqrt[۱۲]{۳^۳} = \sqrt[۱۲]{۲^{۱۳}} \times \sqrt[۶]{۳^۶ \times ۲} \times \sqrt[۱۲]{۲^۸} = \sqrt[۱۲]{۲^{۱۳} \times ۲^۸} = \sqrt[۱۲]{۳^{۱۲} \times ۲^{۱۲} \times ۳}$

$= \sqrt[۱۲]{۲^{۲۴} \times ۳} = ۳ \times ۴ \sqrt[۱۲]{۱۲} =$

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۵-

$$\frac{3^x(1+3+9+27+81+243)}{3^x(\frac{1}{3}+\frac{1}{9}+1+3+9+27)} = \frac{3^x(364)}{3^x(\frac{364}{3})} = \frac{3^x}{3^x} \times \frac{364\times 3}{364} = 3$$

$$\frac{3^x}{3^x} = \frac{9}{3} \quad \boxed{x=2}$$

۶-

$$(a-b)^4(a+b)^4 = (a^2-b^2)^4 = \left(\sqrt{\sqrt{2}(\sqrt{2}-\sqrt{2})} - \sqrt{\sqrt{2}(\sqrt{2}+\sqrt{2})}\right)^4 = \left(\sqrt{2}(\sqrt{\sqrt{2}-\sqrt{2}} - \sqrt{\sqrt{2}+\sqrt{2}})\right)^4 = 2(\sqrt{2}-\sqrt{2}+\sqrt{2}+\sqrt{2}-2)^2 = 8(2+1-2\sqrt{2}) = 16(2-\sqrt{2}) = t(2-\sqrt{2})$$

$$\boxed{t=16}$$

۷-

$$a = \sqrt[4]{7-4\sqrt{3}} = \sqrt[4]{(2-\sqrt{3})^2} = \sqrt{2-\sqrt{3}} \quad \left(a+\frac{1}{a}\right)^2 - 2 = \left(a^2+\frac{1}{a^2}+2-2\right)$$

$$\left(\frac{a^4+1}{a^2}\right)^2 = \left(\frac{(\sqrt{2-\sqrt{3}})^4+1}{(\sqrt{2-\sqrt{3}})}\right)^2 = \left(\frac{7-4\sqrt{3}+1}{2-\sqrt{3}}\right)^2 = 16$$

$2^t = 16 \to t = 4$

۸-

$$A = \sqrt[9]{2^2\sqrt[3]{4}}\left(\frac{1}{2}\right)^{-\frac{4}{3}} = \sqrt[10]{2^{10}} \times \sqrt[3]{2^4} = \sqrt[10]{2^{30}} = 4 \quad (2^3)^{-\frac{1}{3}} = 8^{-\frac{1}{3}} = \frac{1}{2}$$

۹-

$$a^{-1} = 3\sqrt{3} \quad a^2 = \frac{1}{27} \quad \sqrt[7]{a^{14}} \times \sqrt[7]{a^{15}} = 27 \quad \sqrt[7]{a^1} = 27$$

$$\frac{(\frac{1}{a}-3)}{1+\sqrt{3}} = \frac{3\sqrt{3}-3}{1+\sqrt{3}} = \frac{3(\sqrt{3}-1)}{1+\sqrt{3}} = \frac{3(\sqrt{3}-1)^2}{\sqrt{3}^2-1} = \frac{3(3+1-2\sqrt{3})}{2} = 6-3\sqrt{3}$$

۱۰-

$$(\sqrt{x+a}-\sqrt{x-4})(\sqrt{x+a}+\sqrt{x-4}) = 2(\sqrt{x+a}+\sqrt{x-4})$$

$$\to a+x-x+4 \to \sqrt{x+a}+\sqrt{x-4} = \frac{a}{2}+2 \quad \underbrace{\sqrt{x+a}+\sqrt{x-4}}_{\frac{a}{2}+2} = 2 = \frac{a}{2}$$

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