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۱- $(1+6+10+\ldots+5)(2+2+5+5\ldots+5) = ②$

$7 \times 6$

۲- الف) $\sqrt[4]{\frac{1}{4+\sqrt7}} \times \sqrt[4]{(1+\sqrt7)^2} = \sqrt[4]{\frac{1+7+2\sqrt7}{4+\sqrt7}} = \sqrt[4]{\frac{8+2\sqrt7}{4+\sqrt7}} = \sqrt[4]{\frac{2(4+\sqrt7)}{4+\sqrt7}} = \sqrt[4]{2}$

ب) $\frac{\sqrt2+\sqrt5}{\sqrt{10}+2} \times \frac{\sqrt{10}-2}{\sqrt{10}-2} = \frac{2\sqrt5+5\sqrt2-2\sqrt2-2\sqrt5}{10-4} = \frac{3\sqrt2}{6} = \frac{\sqrt2}{2}$

$\frac{\sqrt2}{2}\left(\sqrt{3-\sqrt5} - \sqrt{3+\sqrt5}\right) = \frac{\sqrt{6-2\sqrt5} - \sqrt{6+2\sqrt5}}{2} = \frac{\sqrt{(\sqrt5-1)^2} - \sqrt{(\sqrt5+1)^2}}{2} = \frac{|\sqrt5-1| - |\sqrt5+1|}{2}$

$= \frac{\sqrt5-1-\sqrt5-1}{2} = \frac{-2}{2} = -1$

۳- الف) $\frac{2\sqrt2+3\sqrt3}{5-\sqrt6} \times \frac{5+\sqrt6}{5+\sqrt6} = \frac{10\sqrt2+2\sqrt{12}+15\sqrt3+3\sqrt{18}}{19} = \frac{10\sqrt2+4\sqrt3+15\sqrt3+9\sqrt2}{19}$

$= \frac{19\sqrt2+19\sqrt3}{19} = \sqrt2+\sqrt3-\frac{2}{\sqrt3-1} = \frac{\sqrt6-\sqrt2+3-\sqrt3-2}{\sqrt3-1} = \frac{\sqrt6-\sqrt2-\sqrt3+1}{\sqrt3-1}$

$= \frac{(\sqrt3-1)(\sqrt2)-(\sqrt3-1)}{\sqrt3-1} = \sqrt2-1$

ب) $\frac{3\sqrt3-1}{4+\sqrt3} \times \frac{4-\sqrt3}{4-\sqrt3} = \frac{12\sqrt3-9-4+\sqrt3}{16-3} = \sqrt3-1$

$\frac{1}{2-\sqrt3} \times \frac{2+\sqrt3}{2+\sqrt3} = 2+\sqrt3 \qquad 2+\sqrt3+\sqrt3-1 = 2\sqrt3+1$

۴- الف) $\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}} - 2 = \sqrt6+2-2 = \sqrt6$

$A^2 = \frac{1+\sqrt3+\sqrt3-1+2\sqrt{3-1}}{\sqrt3-\sqrt2} = \frac{2\sqrt3+2\sqrt2}{\sqrt3-\sqrt2} \times \frac{\sqrt3+\sqrt2}{\sqrt3+\sqrt2} = 6+2\sqrt6+2\sqrt6+4 = 10+4\sqrt6$

$= (\sqrt6+2)^2 \longrightarrow A = \sqrt6+2$

ب) $\sqrt[12]{2^8} \times \sqrt[6]{162} \times \sqrt[12]{2^{13}} = \sqrt[12]{2^{13}} \times \sqrt[6]{3^4 \times 2} \times \sqrt[12]{2^8} = \sqrt[12]{2^{13} \times 2^8 \times 3^{12} \times 2^3} = \sqrt[12]{2^{24} \times 3^{12}}$

$= 3 \times 4 = \boxed{12}$

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$\frac{۳^x(۱+۳+۹+۲۷+۸۱+۲۴۳)}{۲^x(\frac{۱}{۴}+\frac{۱}{۲}+۱+۲+۴+۸)} = \frac{۳^x(۳۶۴)}{۲^x(\frac{۶۳}{۴})} = \frac{۳^x}{۲^x} \times \frac{۳۶۴\times۴}{۶۳} = ۵۲$ -۵

$\frac{۳^x}{۲^x} = \frac{۹}{۴}$ $\boxed{x=۲}$

-۶

$(a-b)^۴(a+b)^۴ = (a^۲-b^۲)^۴ = \left(\sqrt{\sqrt۲(\sqrt۳-\sqrt۲)} - \sqrt{\sqrt۲(\sqrt۳+\sqrt۲)}\right)^۴ = \left(\sqrt[۴]{۲}\left(\sqrt{\sqrt۳-\sqrt۲} -\right.\right.$

$\left.\left.\sqrt{\sqrt۳+\sqrt۲}\right)\right)^۴ = ۲(\sqrt۳-\sqrt۲+\sqrt۳+\sqrt۲-۲)^۲ = ۸(۳+۱-۲\sqrt۳) = ۱۶(۲-\sqrt۳) = t(۲-\sqrt۳)$

$\boxed{t=۱۶}$

$a = \sqrt[۴]{۷-۴\sqrt۳} = \sqrt[۴]{(۲-\sqrt۳)^۲} = \sqrt{۲-\sqrt۳}$ $\left(\left(a+\frac{۱}{a}\right)^۲-۲\right)^۲ = \left(a^۲+\frac{۱}{a^۲}+۲-۲\right)^۲$ -۷

$\left(\frac{a^۴+۱}{a^۲}\right)^۲ = \left(\frac{(\sqrt{۲-\sqrt۳})^۴+۱}{(\sqrt{۲-\sqrt۳})}\right)^۲ = \left(\frac{۷-۴\sqrt۳+۱}{۲-\sqrt۳}\right)^۲ = ۱۶$

-۸

$A = \sqrt[۵]{۲^۲\sqrt[۳]{۴}}$ $\left(\frac{۱}{۲}\right)^{-\frac{۴}{۳}} = \sqrt[۱۵]{۲^{۱۰}} \times \sqrt[۳]{۲^۴} = \sqrt[۱۵]{۲^{۳۰}} = ۴$ $(۲A)^{-\frac{۱}{۳}} = ۸^{-\frac{۱}{۳}} = \frac{۱}{۲}$

$\sqrt[۷]{a^{-۱}} = ۲۷$ $\sqrt[۷]{a^{۱۵}} \to ۱ = ۲۷\times\sqrt[۷]{a^{۱۴}} \to \frac{۱}{۲۷} = a^۲$ $a^{-۱} = ۳\sqrt۳$ -۹

$\frac{(\frac{۱}{a}-۳)}{۱+\sqrt۳} = \frac{۳\sqrt۳-۳}{۱+\sqrt۳} = \frac{۳(\sqrt۳-۱)}{۱+\sqrt۳} = \frac{۳(\sqrt۳-۱)^۲}{\sqrt۳^۲-۱} = \frac{۳(۳+۱-۲\sqrt۳)}{۲} = ۶-۳\sqrt۳$

$(\sqrt{x+a}-\sqrt{x-۴})(\sqrt{x+a}+\sqrt{x-۴}) = ۲(\sqrt{x+a}+\sqrt{x-۴}$ -۱۰

$\to a+x-x+۴ \to \sqrt{x+a}+\sqrt{x-۴} = \frac{a}{۲}+۲$ $\underbrace{\sqrt{x+a}+\sqrt{x-۴}}_{\frac{a}{۲}+۲}-۲ = \frac{a}{۲}$