

$$\begin{aligned}
 x^2 - 5x + 2 &= 0 & \alpha, \beta \rightarrow \text{ریشه ها} & \frac{x^2 + px + q}{x^2} & x^2 - 5x + 2 = 0 & \alpha + \beta = -\frac{b}{a} = 5 \\
 \beta^2 = 5\beta - 2 & \rightarrow \beta = 5 - \frac{2}{\beta} & & & \beta^2 - 5\beta + 2 = 0 & \alpha\beta = \frac{c}{a} = 2 \\
 \alpha(5 - \beta) + (5\beta - 2) &= 5\alpha\beta - 19 & & & & \alpha + \beta = 5 \rightarrow \alpha = 5 - \beta \\
 \alpha(5 - \beta) + (5\beta - 2) &= 5\alpha\beta - 19 & & & & \frac{x^2 + px + q}{x^2} = \frac{5(5\beta - 2) - 19}{5\beta - 2} = \frac{25\beta - 38}{5\beta - 2} \\
 \beta &= \frac{5 + \sqrt{25 - 4}}{2} = \frac{5 + \sqrt{17}}{2} & \alpha &= \frac{5 - \sqrt{17}}{2} & & \frac{95(\frac{5 + \sqrt{17}}{2}) - 38}{5(\frac{5 + \sqrt{17}}{2}) - 2} = \frac{299 + 95\sqrt{17}}{21 + 5\sqrt{17}} \xrightarrow{\text{توزیع}} \frac{299 + 95\sqrt{17}}{21 + 5\sqrt{17}} \\
 \frac{299(21 - 5\sqrt{17}) + 95\sqrt{17}(21 - 5\sqrt{17})}{441 - 25 \cdot 17} &= \frac{304}{14} = 19 & \text{جواب} &= 19 & &
 \end{aligned}$$

$$\begin{aligned}
 (3, -4) \quad (-1, 5) & \quad \frac{3 + (-1, 5)}{2} = 0, 75 & \text{نقطه میانه است بین دو رأس پس میانه موازی است.} \\
 \frac{-b}{2a} = \frac{-5}{2} = -2, 5 & \quad \text{جواب} = 1, 5 &
 \end{aligned}$$

$$\begin{aligned}
 |\alpha - \beta| &= \frac{\sqrt{\Delta}}{|a|} & x^2 + 2kx + 5 &= 0 & |\alpha - \beta| &= \frac{2}{k} \\
 k &= \sqrt{4k^2 - 20} \rightarrow 4k^2 - 20 = 16 \rightarrow 4k^2 = 36 \rightarrow k^2 = 9 \rightarrow k = \pm 3 & & & \frac{2}{k} = \frac{\sqrt{4k^2 - 20}}{1} \rightarrow \frac{2}{k} = \sqrt{4k^2 - 20} \\
 \left[\frac{k}{2}\right] &= \left[\frac{9}{2}\right] = [4, 5] & \text{جواب} &= 4 &
 \end{aligned}$$

$$\begin{aligned}
 A(3, 9) \quad B(-5, 9) & \quad \text{نقطه میانه} = \frac{1}{k} & x^2 + \beta^2 = 5 & \rightarrow s^2 - 2p \\
 y = a(x - h)^2 + k & \quad \alpha(x - h)^2 + 1 = a(-5 - h)^2 + 1 & \rightarrow (x - h)^2 &= (-5 - h)^2 \rightarrow 9 - 2h + h^2 = 25 + h^2 + 10h \\
 14h = -14 & \rightarrow h = -1 & y = a(x + 1)^2 + 1 & \rightarrow y = ax^2 + 2x + a + 1 \\
 s^2 - 2p = 5 & \rightarrow \frac{s^2 - 2a}{a} = -2 & p = \frac{a+1}{a} & \rightarrow x - 2\left(\frac{a+1}{a}\right) = 5 \rightarrow -2\left(\frac{a+1}{a}\right) = 1 \\
 -a = 2a + 2 & \rightarrow 3a = -2 \rightarrow a = -\frac{2}{3} & \text{نقطه میانه} = a + 1 & \rightarrow \frac{-2}{3} + 1 = \frac{1}{3} \\
 \text{جواب} &= \left(0, \frac{1}{3}\right) &
 \end{aligned}$$

$$\begin{aligned}
 \Delta > 0 & \quad -x^2 + 5x + m = 0 & \frac{c}{a} = \frac{-m}{1} = -m & \quad \frac{-b}{a} = \frac{5}{1} = 5 \\
 25 - 4ac > 0 & \rightarrow 25 + 4m > 0 \rightarrow 4m > -25 \rightarrow m > -\frac{25}{4} & & (\alpha - \frac{9}{r})(\beta - \frac{9}{r}) > 0 \\
 \alpha\beta - \frac{9}{r}(\alpha + \beta) + \frac{81}{r^2} > 0 & \rightarrow -m - \frac{9}{r}(5) + \frac{81}{r^2} > 0 & \rightarrow m < \frac{9}{r} & \rightarrow m = \frac{-12}{2} = -6 \\
 \text{جواب} &= \text{مجموع} & m = \frac{-25}{4} = -6, 25 & \quad m = \frac{-9}{2} = -4, 5 & \quad m = \frac{-14}{2} = -7
 \end{aligned}$$

$$\begin{aligned}
 y = mx^2 - 12m + 5m - 1 & \quad \text{نقطه میانه} & \frac{-\Delta}{2a} = 2 & \rightarrow 12m = -5 \rightarrow 12m = 20m^2 - 4m - 14m \\
 20m^2 - 12m - 14m = 0 & \rightarrow 20m^2 - 26m = 0 \rightarrow m(20m - 26) = 0 & \rightarrow (m - 1, 3)(m + 1, 2) &= 0 \\
 m = 1, 3 & \quad m = \frac{-12}{2} = -6 & \text{جواب} &= 2 &
 \end{aligned}$$

$$\begin{aligned}
 x^2 + 2(a+1)x + 2a - 1 &= 0 & \alpha, \beta \rightarrow \text{ریشه ها} & \alpha, a, \beta \rightarrow \text{نقطه میانه} & \rightarrow ac = b^2 \\
 \Rightarrow a^2 = \alpha\beta & \quad \alpha\beta = \frac{c}{a} = \frac{2a-1}{1} = 2a-1 & & & a^2 = 2a-1 \rightarrow a^2 - 2a + 1 = 0 \rightarrow (a-1)^2 = 0 \rightarrow a-1 = 0 \\
 x^2 + 2x + 1 &= 0 & \text{جواب} &= a = 1 &
 \end{aligned}$$

$$x^2 - 7x - 5 = 0 \rightarrow t^2 - 7t - 5 = 0 \rightarrow \frac{7 \pm \sqrt{49 + 20}}{2} = \frac{7 \pm \sqrt{69}}{2}$$

$$t_1 = \frac{7 + \sqrt{69}}{2} \quad t_2 = \frac{7 - \sqrt{69}}{2} \quad x = \frac{7 + \sqrt{69}}{2}$$

$$2p^2 - 5p + 2 = 0 \rightarrow p = \frac{5 \pm \sqrt{25 - 16}}{4} = \frac{5 \pm 3}{4}$$

چون گزینه‌های صحیح پس جمع آن‌ها منفی شود.
چون در صورتی که در این معادله از این می‌روند.

$$p^2 = \frac{11 \pm \sqrt{121 - 48}}{2} \rightarrow 2p^2 = 2x \cdot \frac{11 \pm \sqrt{121 - 48}}{2} = \frac{11 \pm \sqrt{121 - 48}}{2} = \frac{11 \pm \sqrt{73}}{2} = \boxed{59 + 7\sqrt{69}} \text{ جواب}$$

$$y = kx^2 - 2x - 2 \quad y = -2x - 2 \quad x^2 - \frac{2}{k}x = \frac{1}{k}(y + 2) \rightarrow (x - \frac{1}{k})^2 = \frac{1}{k^2} \rightarrow x = \frac{1}{k}$$

$$\rightarrow \frac{-b}{2a} = \frac{2}{2k} = \frac{1}{k} \quad (x - \frac{1}{k})^2 = \frac{1}{k^2} \rightarrow x = \frac{1}{k} \pm \frac{1}{k} \rightarrow \frac{-2 - 2k}{k} = \frac{-2}{k} - 2 \rightarrow \frac{-2}{k} - 2 = \frac{-2}{k} - 2$$

$$\rightarrow \frac{-2}{2k} = \frac{-1 - 2k}{k} = \frac{-1 - 2k}{k} \quad \frac{2}{k} = 2 \rightarrow 2k = 2 \rightarrow k = 1 \quad \frac{-2 - 2 \times 2}{2} = \frac{-6}{2} = \boxed{-3} \text{ جواب}$$

$$y = -mx^2 + mx + 1$$

$$y = -m - x$$

$$-mx^2 + mx + 1 = -m - x$$

$$mx^2 - mx - m - x - 1 = 0 \rightarrow mx^2 - (m+1)x - m - 1 = 0 \rightarrow \Delta < 0$$

$$b^2 - 4ac < 0 \rightarrow (m+1)^2 - 4(m)(-m-1) < 0 \rightarrow m^2 + 1 + 2m + 4m^2 + 4m < 0$$

$$5m^2 + 5m + 1 < 0 \rightarrow m^2 + m + \frac{1}{5} < 0 \rightarrow (m + \frac{1}{2})(m + \frac{1}{5}) < 0 \rightarrow (\frac{5m+1}{5})(m + \frac{1}{5}) < 0$$

$$-\frac{1}{5} < m < -1$$

به ازای هیچ مقدار عدد صحیح
صفر

