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پرنیان دهقانی

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$\alpha+\beta=5 ,\ \alpha\beta=2$

$x^2=5x-2 \rightarrow \beta^2=5\beta-2 \rightarrow \beta^3=\beta(5\beta-2) = 5\beta^2-2\beta = 5(5\beta-2)-2\beta$

$25\beta-10-2\beta = 23\beta-10$

$\beta^4 = \beta(\beta^3) = \beta(23\beta-10) = 23\beta^2-10\beta$

$\hookrightarrow 23(5\beta-2)-10\beta = 105\beta-46-10\beta = 95\beta-46$

۵

$\beta^5=\beta(\beta^4) = \beta(95\beta-46) = 95\beta^2-46\beta = 95(5\beta-2\beta) = 429\beta-210$

$4\alpha+\beta^5 = 4\alpha+429\beta-210 = 4(5-\beta)+429\beta-210 = 20-4\beta+429\beta-210 = 425\beta-190 = 5(95\beta-38)$

$$\boxed{\frac{\cancel{\alpha}(95\beta-38)}{\cancel{\alpha}\beta^2} = \frac{95\beta-38}{\beta^2} = \frac{95}{\beta}-\frac{38}{\beta^2}}$$

$\frac{1}{\beta}=\frac{\alpha}{2} \rightarrow \frac{1}{\beta^2}=\frac{\alpha^2}{4}$ $\quad 95\times\frac{1}{\beta}-38\times\frac{1}{\beta^2} = 95\times\frac{\alpha}{2}-38\times\frac{\alpha^2}{4} = \frac{1}{2}(95\alpha-19\alpha^2) \xrightarrow{\alpha^2=5\alpha-2} 5\alpha-\alpha^2=2$

$\frac{1}{2}\times 19(5\alpha-\alpha^2) = \frac{1}{2}\times 19\times 2 = \underline{19}$

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۲- محور تقارن سهمی $= x = \frac{3-1.25}{2} = \frac{1.75}{2} = 0.875 \rightarrow$ محور تقارن $=$ $\frac{\text{مجموع ریشه‌ها}}{2} \rightarrow \frac{\alpha+\beta}{2} = 0.875 \rightarrow \alpha+\beta=1.75$

۳-

$ax^2+bx+c=0 \rightarrow \alpha+\beta = \frac{-2k}{1} = -2k$ $\quad \alpha=m+d \quad \beta=m-d$

$|\alpha-\beta| = 2d$ $\quad 2d=\frac{4}{3}k \rightarrow d=\frac{2}{3}k \rightarrow \alpha = -k+\frac{2}{3}k = -\frac{k}{3} \ ,\ \beta = -k-\frac{2}{3}k = -\frac{5}{3}k$

$(-\frac{k}{3})^2+2k(-\frac{k}{3})+5=0$ $\quad \frac{k^2}{9}-\frac{2k^2}{3}+5=0$ $\quad \frac{k^2}{9}-\frac{6k^2}{9}+5=0$ $\quad -\frac{5k^2}{9}+5=0 \rightarrow 5=\frac{5k^2}{9} \Rightarrow k^2=9$

۳

$[\frac{k^2}{2}] = [\frac{9}{2}] = 4$

۴-

x رأس $= \frac{3+(-5)}{2} = -1$
$\hookrightarrow \alpha+\beta=-2$

$y=a(x+1)^2+1$ $\quad \alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta = 4-2\alpha\beta$

$\alpha\beta = 1-d^2 = 1-(-\frac{1}{a}) = 1+\frac{1}{a}$

۴

$\alpha^2+\beta^2 = 4-2(1+\frac{1}{a}) = 2-\frac{2}{a}$

$2-\frac{2}{a}=5 \rightarrow -\frac{2}{a}=3 \rightarrow a=-\frac{2}{3}$

$y=-\frac{2}{3}(x+1)^2+1$

$y(0) = -\frac{2}{3}\times 1+1 = 1-\frac{2}{3} = \frac{1}{3}$

$\dfrac{-5 \pm \sqrt{25-4(-1)(m)}}{-2} \leq \dfrac{9}{2} \rightarrow \pm\sqrt{25+4m} < 4$ ۵

$x^2+5x+m=0 \longrightarrow x_2+x_1=5 \;,\; x_1x_2=m$ باید $\begin{cases} \Delta>0 \rightarrow 5^2-4m = 25-4m \geq 0 \rightarrow m \leq 6 \\ x=4.5 \rightarrow y = \text{مثبت} \rightarrow f(4.5)>0 \end{cases}$ — ۵

$0 \leq 25+4m < 16$

$-6.25 < m < -2.25$ $m \in \{-3,-4,-5,-6\}$

~~$f(4.5) = (4.5)^2 + 5\times4.5+m = 42.75+m>0 \rightarrow m>-42.75$~~ $\longrightarrow -42 \leq m \leq 6 \longrightarrow$ ۴۹ مقدار

$y = mx^2-12x+5m-1 \longrightarrow Min \longrightarrow m>0$ — ۶

$x_{\text{راس}} = \dfrac{-b}{2a} = \dfrac{-12}{-2m} = \dfrac{6}{m}$ $y_{min} = m\left(\dfrac{6}{m}\right)^2 - 12x+5m-1 = \dfrac{36}{m} - \dfrac{72}{m} + 5m - 1 = -\dfrac{36}{m}+5m-1$

$-\dfrac{36}{m}+5m-1 = 2 \longrightarrow 5m - \dfrac{36}{m} = 3 \rightarrow 5m^2-36 = 3m \rightarrow 5m^2-3m-36=0 \rightarrow m = \dfrac{3\pm\sqrt{9+720}}{10} = \dfrac{3\pm\sqrt{729}}{10}$

$m = \dfrac{3+\sqrt{729}}{10}$ / $m_2 = \dfrac{3-\sqrt{729}}{10} \rightarrow$ غ ق ق چون $m>0$ $x = \dfrac{6}{m} = \dfrac{6}{3} = 2$

$\dfrac{3+27}{10} = 3$

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$\alpha+\beta = -2(a+1)$ $\alpha\beta = 2a-1$ $\alpha, a, \beta \longrightarrow a^2 = \alpha\beta$ — ۷

$a^2 = 2a-1$

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$a^2-2a+1 = 0 \rightarrow (a-1)^2 = 0 \rightarrow \boxed{a=1}$

$x^4-7x^2-5=0 \longrightarrow x^2=t \longrightarrow \underset{\text{جمع ریشه‌ها}}{t^2-7t}\;\underset{\text{ضرب ریشه}}{-5} = 0 \longrightarrow \cancel{2p^2-3sp}$ $S=-7$ $P=-5$ — حل در پشت صفحه — ۸

~~$2\times25-3\times(-7\times-5)-14$~~

~~$50-105-14 = -69$~~

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$y = kx^2-4x+6$ $x_{\text{راس}} = \dfrac{-b}{2a} = \dfrac{-4}{-2k} = \dfrac{2}{k}$ ✓   ~~$y_{\text{راس}} = k\left(\dfrac{2}{k}\right)^2 - 4\times\dfrac{2}{k}+6 = \dfrac{4}{k} - \dfrac{8}{k} + 6 = -\dfrac{4}{k}+6$~~ — ۹

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~~$y_{\text{راس}} = \dfrac{-\Delta}{4a} = \dfrac{-(16-24k)}{4k} = \dfrac{-16+24k}{4k} = \dfrac{6k-4}{k}$~~ $S\left(\dfrac{2}{k}, \dfrac{6k-4}{k}\right)$

$\dfrac{6k-4}{k} = -4\left(\dfrac{2}{k}\right)-4$

~~$\dfrac{6k-4}{k} = \dfrac{-8}{k}-4 \longrightarrow \dfrac{6k-4}{k} = \dfrac{-8-4k}{k} \rightarrow 6k-4 = -8-4k \longrightarrow 10k = -4 \longrightarrow k = -0.4$~~

$\rightarrow K(x_1)^2 - 4(x_1) - 6$

$\rightarrow -4 = K(x_1)^2 - 6 \rightarrow 2 = K\left(\dfrac{2}{K}\right)^2 \rightarrow K = 2$

$\begin{cases} x_1 = 1 \\ y_1 = -8 \end{cases}$

$-mx^{۲}+mx+۱=-m-x$

$-mx^{۲}+mx+۱+m+x=۰$

$-mx^{۲}+(m+۱)x+۱+m=۰ \xrightarrow{\text{ریشه نداشته باشد}} (m+۱)^{۲}-۴(m+۱)(-m)<۰$

$-۴(-m^{۲}-m)$

$=۴m^{۲}+۴m$

$۴(m^{۲}+m)$

$(m+۱)^{۲}-۴m(m+۱)$

$(m+۱)\underbrace{(m+۱-۴m)}_{(-۳m+۱)}<۰$

$۱)\ (-۳m+۱)>۰$ و $(m+۱)<۰$

$-۳m<۱ \rightarrow m<\frac{۱}{۳}$ ، $m<-۱$ $\rightarrow -\frac{۱}{۳}<m<-۱$

~~شامل $\frac{۱}{۳}$ عدد صحیح است~~

~~که $m=۰$ است~~

$۲)\ (-۳m+۱)<۰$ و $(m+۱)>۰$

$-۳m<-۱ \rightarrow ۳m<۱ \rightarrow m<\frac{۱}{۳}$ ، $m>-۱$ $\rightarrow$ ~~$-۱<m<\frac{۱}{۳}$~~

$۵m^{۲}+۶m+۱<۰$ ۰,۵   ۱

$(m+۱)(۵m+۱)<۰$

$-۱<m<\frac{-۱}{۵}$

سوال ۸

$t=\frac{۷\pm\sqrt{۶۹}}{۲} \rightarrow x^{۲}=\frac{۷+\sqrt{۶۹}}{۲}$

$x=\pm\sqrt{\frac{۷+\sqrt{۶۹}}{۲}} \rightarrow S=۰$ و $P=-\frac{۷+\sqrt{۶۹}}{۲}$

$۲P^{۲}-۳SP+۲S=۲\left(-\frac{۷+\sqrt{۶۹}}{۲}\right)^{۲}=$

$\frac{۱}{۲}(۴۹+۶۹+۱۴\sqrt{۶۹})=۵۹+۷\sqrt{۶۹}$