$x^2 - 5x + 2 = 0 \quad \frac{-b}{a} = 5 = \alpha + \beta \quad \frac{c}{a} = \alpha\beta = 2$

(1

$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta = 10 \rightarrow \alpha^2 + \beta^2 = 21 \rightarrow \beta^2 = (21 - \alpha^2)^2 \rightarrow$

$\frac{4\alpha + \beta^5}{5\beta^2} \Rightarrow \frac{2\alpha^2\beta + \beta^5}{\alpha\beta^2 + \beta^4} \rightarrow \frac{\beta(2\alpha^2 + \beta^4)}{\beta(\alpha\beta + \beta^3)} = \frac{\beta(\alpha^4 + \beta^4)}{5\beta} \rightarrow$ 19

$\frac{-b}{2a} = \frac{4-1,5}{2} = 0,75 \rightarrow \frac{3}{4}$ $\quad (\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta) = \alpha^3+\beta^3$

$\frac{5}{5}$ $\quad$ 19

(2

$K^2 = \frac{361,\ldots}{2} \simeq 15$

$x^2 + 2Kx + 5 = 0 \quad \left[\frac{K^2}{2}\right] = ? \quad -2K = \alpha + \beta$

(3

$5 = \alpha\beta$

$\frac{4}{3}K = \alpha - \beta$

$-2K = \alpha + \beta \rightarrow -2K = \frac{-2}{3}K + \beta \rightarrow \beta = \frac{-4}{3}K$

$\frac{-2}{3}K = \alpha \qquad \frac{-8}{3}K \times \frac{-2}{3}K = 5 \rightarrow \frac{16}{9}K = 5 \rightarrow K = \frac{3\sqrt{5}}{4} \simeq 5,\ldots$

(4

$\frac{-\Delta}{8a} = 1 \qquad B = (-5, y) \Longrightarrow y = ax^2 + bx + c$

$A = (3, y)$

$y = ax^2 + bx + c$

$-4a = \Delta$

$9a - 3b + c = 9a + 3b + c \rightarrow 19a = 15b$

$4a = b \rightarrow \frac{-b}{a} = -4$

$-8a = b - 4ac$

$8a^2 - 8ac = -8a \rightarrow a - c = -1 \rightarrow a - 1 = c$

$\alpha^2 + \beta^2 = 5 \rightarrow (\alpha+\beta)^2(-2)^2 \rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 4 \rightarrow 2\alpha\beta = -1$

$\alpha\beta = \frac{-1}{2}$

$\frac{-1}{2} = \frac{c}{c+1} \qquad -c - 1 = 2c \qquad 3c = -1 \rightarrow c = \frac{-1}{3}$

۵)

$-x^2+5x+m=0$ $\quad +5=\alpha+\beta$ $\quad -m=\alpha\beta$

ریشه‌ها α و β مثبت

$+\ \beta \neq \alpha \neq$

$m>\frac{81}{4}$

$\alpha<\frac{9}{2}$, $\beta<\frac{9}{2}$ $\Big\}\ \alpha\beta<\frac{81}{4}$

۶)

$y=mx^2-12x+5m-1 \to \frac{-\Delta}{4a}=2 \to -\Delta=8a$

$-(144-4m(5m-1))=8a$

$144-20m^2+4m=-8a \to 144-16m=8a \to 18-2m=m$

$m=6$

$\frac{-b}{2a}=\frac{12}{18-2m} \longrightarrow x=\frac{12}{36-24}$ $\quad y=2$ $\quad \boxed{x=1}$

۷)

$x^2+2(a+1)x+2a-1=0$ $\quad a^2=\frac{\alpha\beta}{2a-1} \longrightarrow a^2-2a+1=0$

$2a-1=\alpha\beta$ $\quad \boxed{a=1}$

۸)

$x^4-7x^2-5=0$ $\quad (x^2=t)$

$t^2-7t-5=0 \to 49+20=69 \to \frac{7+\sqrt{69}}{2}=x^2$

$x=\pm\sqrt{\frac{7+\sqrt{69}}{2}} \longrightarrow S=0$ $\quad 2P^2-3SP+2S$

$P=\frac{-7-\sqrt{69}}{2}$

$\boxed{59+7\sqrt{69}}=\frac{118+14\sqrt{69}}{2}$ $\quad 2\left(\frac{49+69+14\sqrt{69}}{4}\right)=$

۹)

$y=kx^2-4x-6$ $\quad y=-4x-4$ $\quad \frac{4+6k}{k}=\frac{-8-4k}{k}$

$x_{راس}=\frac{4}{2k}=\frac{2}{k}$ $\quad y_{راس}=\frac{-\Delta}{4a}=\frac{16+24k}{4k} \Rightarrow \frac{4+6k}{k} \to \frac{\frac{20-36}{5}}{\frac{-6}{5}}$, $\frac{-16}{-6}$

$4+6k=-8-4k \to 12=-10k \to k=-\frac{6}{5}$

(۱۰

$y = -mx^2 + mx + 1$

$y = -m - x$

$-mx^2 + mx + 1 = -m - x$

$-mx^2 + mx + 1 + m + x = 0$

$-mx^2 + (m+1)x + 1 + m = 0$

$\Delta < 0 \rightarrow m^2 + 1 + 2m + 4m(1+m) < 0$

$m^2 + 1 + 2m + 4m + 4m^2 < 0$

$5m^2 + 6m + 1 < 0$

$-\frac{1}{5} \quad -1$

$-1 \quad -\frac{1}{5}$

$-1 < m < -\frac{1}{5}$