

$$\frac{\alpha^2 + \beta^2}{\alpha\beta^2} = \frac{\alpha^2}{\alpha\beta^2} + \frac{\beta^2}{\alpha\beta^2} = \frac{\alpha^2}{\alpha\beta^2} + \frac{\beta^2}{\alpha} \left\{ \begin{array}{l} \frac{\alpha^2}{\alpha(\frac{\alpha}{\alpha})^2} + \frac{\beta^2}{\alpha} = \frac{\alpha^2}{\alpha} + \frac{\beta^2}{\alpha} \rightarrow \alpha^2 + \beta^2 = 5^2 = 25 \\ \sim \frac{\alpha^2 - 4 \times 2 \times \alpha}{\alpha} = \frac{9\alpha}{\alpha} = 9 \end{array} \right.$$

$P = \alpha\beta = 2 \rightarrow \beta = \frac{2}{\alpha}$
 $5 = \alpha + \beta = \alpha$

$$y = ax^2 + bx + c \quad y = 4, \quad 21 \times 2a - 11b + c = 9a + 2b + c \rightarrow 41 \times 2a - 11b + c = 0$$

$$\rightarrow b = 11a \quad \text{مجموع ریشه ها} = -\frac{b}{a} = -\frac{11a}{a} = -11a$$

$$D = \frac{\Delta}{a} = \frac{c}{a} k^2 \rightarrow \frac{\Delta}{a^2} = \frac{14}{9} k^2 \rightarrow \frac{c k^2 - 20}{9} = \frac{14}{9} k^2 \rightarrow 4k^2 - 11a = 14k^2$$

$$\rightarrow 20k^2 - 11a = 0 \rightarrow 20(k^2 - 9) = 0 \rightarrow k^2 - 9 = 0 \rightarrow k^2 = 9$$

$$\left[\frac{k^2}{9} \right] = \left[\frac{9}{9} \right] = [1] = \overline{f}$$

$$x_5 = \frac{-a+2}{2} = -1 \rightarrow \text{مطلوبه} \rightarrow \frac{-b}{2a} = -1 \rightarrow \left[\frac{b}{a} = 2 \right] \textcircled{1}$$

$$\alpha^2 + \beta^2 = a \rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = a + 2\alpha\beta \rightarrow (\alpha + \beta)^2 = a + 2\alpha\beta = a + 2P$$

$$2P = -1 \rightarrow P = -\frac{1}{2} = \frac{c}{a} \textcircled{2}$$

$$ax^2 + bx + c = y \rightarrow a(x^2 + \frac{b}{a}x + \frac{c}{a}) = y \xrightarrow{\textcircled{1}, \textcircled{2}, \textcircled{3}} a(1 - 2 - \frac{1}{2}) = 1 \rightarrow a = -\frac{2}{3} \textcircled{4}$$

$$\frac{c}{a} = -\frac{1}{2} \textcircled{5} \rightarrow c = \frac{1}{3} \rightarrow$$

$$\Delta_{\pm} = \frac{-a \pm \sqrt{a^2 - 4cm}}{2} \Rightarrow \left(\frac{-a - \sqrt{a^2 - 4cm}}{-2m} < \frac{9}{2} \right) \times 2 \rightarrow a + \sqrt{a^2 + 4cm} < 9$$

$$\rightarrow \sqrt{a^2 + 4cm} < 9 - a \rightarrow a^2 + 4cm < (9 - a)^2 \rightarrow 4cm < -9 - 2a \rightarrow m < -\frac{9}{4} \rightarrow m < -2.25$$

$$\Delta > 0 \rightarrow a^2 + 4cm > 0 \rightarrow m > -\frac{a^2}{4} \rightarrow m > -1.125$$

$$-4.125 < m < -2.25 \rightarrow \underline{-4, -2, -3} \rightarrow \underline{3}$$

$$y = mx^2 - 12x + 2m - 1 \Rightarrow \min_{x \in \mathbb{R}} y \rightarrow m < 0, \text{ عرضی } x = -\frac{b}{2a} = -\frac{y}{m}, \text{ عرضی } y = y \rightarrow y_{\min}$$

$$y = m\left(\frac{y}{m}\right)^2 - 12\left(\frac{y}{m}\right) + 2m - 1 \rightarrow y = 2m - \frac{12y}{m} - 1 \Rightarrow 2m^2 - 12m - y = 0$$

$$\Delta = 144 - 4(2)(-y) = 144 + 8y \rightarrow m_{1,2} = \frac{12 \pm \sqrt{144 + 8y}}{4} = 3 \pm \frac{\sqrt{144 + 8y}}{2}, \frac{y}{m} = 2 \rightarrow m = 3$$

$$\text{عرضی } x = \text{عرضی } y = \frac{y}{m} \xrightarrow{m=3} \frac{y}{3} = 2 \rightarrow \text{مختار: } x = 2$$

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$$x^2 + x(a+1) + 2a - 1 = 0, \alpha, \beta \rightarrow \alpha^2 + \alpha\beta + \alpha^2 = \frac{c}{a} \rightarrow \alpha^2 = 2a - 1$$

$$-\alpha^2 - 2a + 1 = 0 \rightarrow (\alpha - 1)^2 = 0 \rightarrow \alpha = 1 \text{ (۱)}, \Delta > 0 \text{ (۲)}$$

$$\text{(۱), (۱)} \rightarrow x^2 + x(1+1) + 2(1) - 1 = 0 \rightarrow x^2 + 2x - 1 = 0 \rightarrow \Delta = 14 - 4(1)(-1) > 0 \checkmark \text{ (۳)}$$

$$\text{(۱), (۲), (۳)} \rightarrow \boxed{\alpha = 1}$$

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$$x^2 = t \rightarrow t^2 - vt - 2 = 0 \rightarrow t_{1,2} = \frac{v \pm \sqrt{v^2 + 8}}{2} \rightarrow x^2 = \frac{v \pm \sqrt{v^2 + 8}}{2} \Rightarrow x = \pm \sqrt{\frac{v \pm \sqrt{v^2 + 8}}{2}}$$

$$S = 0 \rightarrow x_S, x_P = 0 \rightarrow x_P^2 = x \left[\left(-\sqrt{\frac{v + \sqrt{v^2 + 8}}{2}} \right) \left(\sqrt{\frac{v + \sqrt{v^2 + 8}}{2}} \right) \right]^2 = x \left(\frac{-v + \sqrt{v^2 + 8}}{2} \right)^2$$

$$\Rightarrow x \left(\frac{v^2 + 1(v^2 + 8)}{4} \right) = \frac{11v + 1\sqrt{v^2 + 8}}{2} \Rightarrow x_P^2 = \frac{2v^2 + v\sqrt{v^2 + 8}}{2}$$

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$$y = kx^2 - 6x - 4, \text{ عرضی } x = -\frac{b}{2a} = -\frac{y}{k}, \text{ عرضی } y = k\left(\frac{y}{k}\right)^2 - 6\left(\frac{y}{k}\right) - 4 \rightarrow \text{عرضی } y = -\frac{6}{k} - 4$$

$$y = -6x - 4 \rightarrow -\frac{6}{k} - 4 = -6\left(\frac{y}{k}\right) - 4 \Rightarrow \frac{6}{k} = 2 \rightarrow k = 3$$

$$kx^2 - 6x - 4 = y \rightarrow \frac{-\Delta}{2a} = \frac{-(14 - 4(3)(-4))}{2(3)} = -\frac{26}{3} \Rightarrow y = -\frac{26}{3}$$

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$$-mx^2 + m\lambda + 1 = -m - \lambda \rightarrow -mx^2 + m\lambda + 1 + m + \lambda \rightarrow \Delta = (m+1)(\lambda m + 1) < 0$$

$$\rightarrow \frac{-1}{+} \mid \frac{-1}{-} \mid \frac{+}{+} \rightarrow m = (-1, -\frac{1}{\lambda}) \rightarrow \text{شماره صحیح عدد صحیح نیست}$$

(چون باید حاصل بر فرد و برابر باشد)
فاصله باشد و این یعنی این $\Delta < 0$

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