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۱) پیشنهادی تکلیف ۲۳ (هم‌زمان)

$\frac{-b}{2a} = -1 \to b = 2a$ ✓ $x=-1 \to a-b+c=9 \to -a+c=9$ ✓

$x=3 \to 9a+3b+c=1 \to 12a=-8 \to a=-\frac{1}{2} \to b=2(\frac{-1}{2})=0$

$c = 9-\frac{1}{2} = \frac{17}{2}$

$y = -\frac{1}{2}x^2 - x + \frac{17}{2}$

۲) ✓

$y=2x^2+mx+m+9=0$ → دوریشه‌ای $\Rightarrow m^2-4(2m+18)>0$

$\to m^2-8m-72>0 \to (m+6)(m-12)>0 \to$

| $x$ | $-\infty$ | $-6$ | $12$ | $+\infty$ |
|---|---|---|---|---|
| $y$ | + | − | + | |

$\Rightarrow$ ① $m \in (-\infty,-6) \cup (12,+\infty)$

② $S = -\frac{m}{2} > 0 \to m < 0$

③ $\frac{m+9}{2} > 0 \to m > -9$

$\cap \to -9 < m < -6$

۳) ✓

$3x^2+(2m-1)x+2-m=0$ → دوریشه $\Rightarrow \Delta = 4m^2+1-4m-4(9-3m) > 0$

$\Rightarrow 4m^2+8m-23>0 \to m = \frac{-8 \pm \sqrt{64-4(4)(-23)}}{8} = \frac{-8 \pm \sqrt{4^2(23)}}{8}$

$= \frac{-2 \pm \sqrt{23}}{2} \Rightarrow$

| $x$ | $-\infty$ | $\frac{-1-\sqrt{23}}{2}$ | $\frac{-1+\sqrt{23}}{2}$ | $+\infty$ |
|---|---|---|---|---|
| $y$ | + | − | + | |

$\Rightarrow$

① $m \in (-\infty, \frac{-1-\sqrt{23}}{2}) \cup (\frac{-1+\sqrt{23}}{2}, +\infty)$

② $\alpha+\beta = \frac{1}{\alpha\beta} \Rightarrow \frac{-2x+1}{3} = \frac{+}{\frac{2-m}{3}} \to 2m^2-3m-7=0$

$(2m-7)(m+1)=0$

$\Rightarrow m \begin{cases} ① -1 \\ ② \frac{7}{2} \end{cases} \cap \to m = \frac{7}{2}$

۴) ۵

$x^2 - x - 4 = 0 \rightarrow S = 1$
$P = -4$

$x_1^3 + x_2^3 + \frac{1}{x_1} + \frac{1}{x_2} = (x_1+x_2)^3 - 3(-4)(x_1+x_2) + \frac{x_1+x_2}{-4}$

$= 1 + 12 + \frac{1}{-4} = \frac{48}{4} - \frac{1}{4} = \frac{47}{4}$

$(x_1^2 + \frac{1}{x_1})(x_2^2 + \frac{1}{x_2}) = x_1^2 x_2^2 + \frac{1}{x_1 x_2} + x_1^2 + x_2^2$

$= (x_1 x_2)^2 + \frac{1}{x_1 x_2} + (x_1+x_2)^2 - 2x_1x_2$

$= 16 - 4 + 1 - 8 = 5$

$\Delta = 1 + 16 = 17$

$y = x^2 - \frac{17}{4}x + 5$

۵) ۵

$\sqrt[3]{x} = t \rightarrow (t + \frac{1}{t})(t-1)^8 = t^2 - \frac{1}{t} \rightarrow \sqrt[3]{x^4} - \frac{1}{\sqrt[3]{x^2}} = \sqrt[3]{x^2}$

$\Rightarrow \frac{\sqrt[3]{x^4}-1}{\sqrt[3]{x^2}} = 2\sqrt[3]{x} \rightarrow x^2 - 1 = 2x \rightarrow x^2 - 2x - 1 = 0$

$S = 2$ ↙ ↘ $-S = -2$

$S = 2$

۶) ۴

$x^2 - \alpha x + 4 = 0 \quad \alpha = 2\beta \rightarrow P = 4 \Rightarrow 2\beta^2 = 4 \rightarrow \beta^2 = 2 \rightarrow \beta^2 = \frac{6}{3}$

$\rightarrow \alpha \rightarrow ① \; 2$
$\rightarrow \beta \; -2 \Rightarrow S \rightarrow ① \; \alpha = 3$
$\rightarrow ② \; \alpha = -1$

$\beta \rightarrow ① \; -3$
$② \; 3$

$\rightarrow 3 - (-1) = 4$

۱۰) ۵

۱) $x^2 + 4x + m = 0 \rightarrow S = -4 \rightarrow \alpha = -4 - \beta$ (ریشه صحیح)

۲) $x^2 + 2x - 3m = 0 \rightarrow S = -2 \rightarrow \alpha = -2 - \beta$

$-4 - \beta = -2 - \beta$
$\beta = 4$

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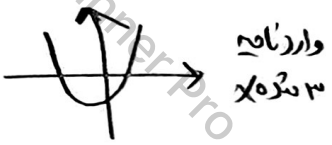
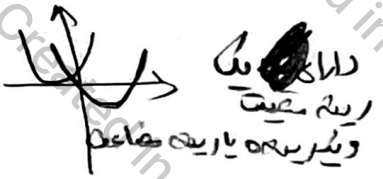
$y = ax^2 + (2+2a)x \Rightarrow$

① $a > 0$

②  وارونه محور $x$ها را در ۳ نقطه قطع می‌کند

 دارای یک ریشه مضاعف و دیگر ریشه‌ها یا ریشه‌های مضاعف

$\Delta \geq 0 \rightarrow 4 + 4a^2 + 12a \geq 0$

$(2a+2)^2 \geq 0 \rightarrow a \neq \frac{-2}{2}$

③ $S > 0 \quad \frac{-2-2a}{a} > 0 \rightarrow -\frac{2}{2} < a$

$\rightarrow$ ۵ هیچ مقدار

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$y = -x^2 - 2x + b \;,\; y = x^2 + ax - 2$

$\hookrightarrow \frac{-b}{2a} = \frac{2}{-2} = \frac{-\alpha}{2} \rightarrow \alpha = 2$

$\begin{cases} 1 = -x^2 - 2x + b \\ 1 = x^2 + 2x - 2 \end{cases} \rightarrow 2 = -2 + b \rightarrow b = 4 \qquad a \times b = 2 \times 4 = 8$

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$2x^2 - ax + b = 0 \rightarrow S = \alpha + \frac{1}{2} + \beta + \frac{1}{2} = -\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

$= \frac{-(-a)}{2} \rightarrow \alpha = 1$

$P = (\alpha + \frac{1}{2})(\beta + \frac{1}{2}) = \alpha\beta + \frac{1}{2}(\alpha+\beta) + \frac{1}{4} = \frac{b}{2}$

$\rightarrow -3 + \frac{1}{2}(-\frac{1}{2}) + \frac{1}{4} = \frac{b}{2} \rightarrow b = -6$

$ab = (-6) \times 1 = -6 \rightarrow \left[\frac{-6}{2}\right] = \left[\frac{-3}{2}\right] = -2$