بنام خدا...

روژینا فدائی

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الف) $y=2x^2-4x+1 \to a>0 \to \underline{min}$ دارد $\left|\begin{matrix}\frac{-b}{2a}\\ \frac{-\Delta}{4a}\end{matrix}\right. \Rightarrow \left|\begin{matrix}\frac{4}{4}=1\\ \frac{-8}{8}=-1\end{matrix}\right. \Rightarrow \left|\begin{matrix}1\\ -1\end{matrix}\right.$

$\Delta=b^2-4ac \Rightarrow 16-(4)(2)(1)=8$

ب) $y=-2x^2+3x-5 \to a<0 \to \underline{max}$ دارد $\left|\begin{matrix}\frac{-b}{2a}\\ \frac{-\Delta}{4a}\end{matrix}\right. \Rightarrow \left|\begin{matrix}\frac{-3}{-4}\\ \frac{31}{-8}\end{matrix}\right. \Rightarrow \left|\begin{matrix}\frac{3}{4}\\ \frac{-31}{8}\end{matrix}\right.$

$\Delta=b^2-4ac \Rightarrow 9-(4)(-2)(-5)=-31$

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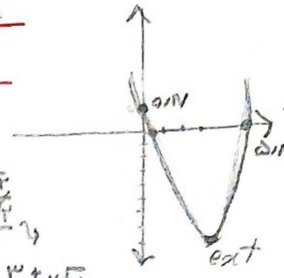
الف) $y=x^2-6x+1 \longrightarrow$ $a>0$ $\underline{min}$; $x=0 \Rightarrow y=1$ $(0,1)$; $ext \Rightarrow \left|\begin{matrix}\frac{b}{2a}\\ \frac{-\Delta}{4a}\end{matrix}\right. \Rightarrow \left|\begin{matrix}\frac{6}{2}=3\\ \frac{-32}{4}=-8\end{matrix}\right.$

$\to y=4x^2-6x+1$

$\Delta=b^2-4ac \Rightarrow 36-(4)(1)(1)=32$

$x=\frac{+6\pm\sqrt{32}}{2} \Rightarrow 3\pm2\sqrt2$

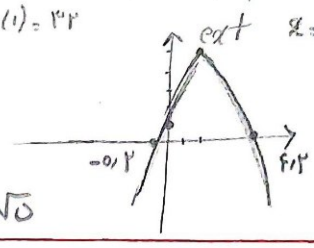


ب) $y=-x^2+4x+1$; $a<0$ $\underline{max}$

$\longrightarrow x=0 \Rightarrow y=1$

$ext \Rightarrow \left|\begin{matrix}\frac{-b}{2a}\\ \frac{-\Delta}{4a}\end{matrix}\right. \Rightarrow \left|\begin{matrix}\frac{-4}{-2}=2\\ 5\end{matrix}\right.$ جا ماندی

$\Delta=b^2-4ac \Rightarrow$
$16-(4)(1)(-1) \Rightarrow 20$
$x=\frac{-4\pm\sqrt{20}}{-2} \Rightarrow 2\pm\sqrt5$ ($\sqrt{20}=2\sqrt5$)



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$4x^3+12x^2-9x-2=0 \to$ چون درجه سوم است تا یک ریشه دیگرم دارد $\Rightarrow 4\underbrace{(x-\beta)(x-\alpha)}(x-c) \Rightarrow$

$\alpha\beta=-2$

$\alpha+\beta=1$

$(4x^2-4x\alpha-4\beta x+4\beta\alpha)(x-c) \Rightarrow 4x^3-4x^2\alpha-4x^2\beta+4x\beta\alpha-4cx^2+4x\alpha c+4\beta cx-4\beta\alpha c$

$\Rightarrow \cancel{4x^3}+12x^2-9x-2=\cancel{4x^3}+x^2(-4\alpha-4\beta-4c)+x(4\beta\alpha+4\alpha c+4\beta c)-4\beta\alpha c$

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$-4(\alpha+\beta+c)=k \Rightarrow -4(1-\frac14)=k \to -4\times\frac34=\boxed{-3=k}$

$4(\beta\alpha+\alpha c+\beta c)=-9$

$-4\beta\alpha c=-2 \longrightarrow 8c=-2 \to c=\frac{-2}{8}=\frac{-1}{4}$

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$x^2-3mx+m=0$

$\sqrt\beta-\sqrt\alpha=1 \to (\sqrt\beta-\sqrt\alpha)^2=1 \to \beta+\alpha-2\beta\alpha=1 \Rightarrow 3m-2m=1 \to \boxed{m=1}$

$\alpha+\beta=\frac{-b}{a}=\frac{3m}{1}=3m$ $\qquad 2x^2-mx-m=0 \Rightarrow \alpha'\beta'=\frac{c'}{a'} \Rightarrow \frac{-m}{2}=\boxed{\frac{-1}{2}}$

$\alpha\beta \to m$

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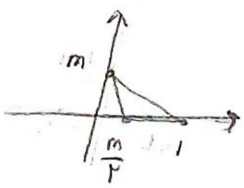
$y=2x^2-(m+2)x+m \xrightarrow[x=0]{\text{تقاطع با عرض}} y=m$ $\qquad \Delta=b^2-4ac \Rightarrow (m+2)^2-4(2)(m) \Rightarrow$

$m^2+4+4m-8m \Rightarrow m^2+4-4m \to (m-2)^2$

$x=\frac{-b\pm\sqrt\Delta}{2a} \Rightarrow \frac{m+2\pm\sqrt{(m-2)^2}}{4}$

$m-2>0 \Rightarrow \frac{m+\cancel{2}+m-\cancel{2}}{4}=\frac{2m}{4}=\frac12 m$ (-1) یا $\frac{\cancel{m}+2-\cancel{m}+2}{4}=1$

$m-2<0$: $\frac{m+2-m+2}{4}=1$ یا $\frac{m+2+m-2}{4}=\frac12 m$



$S=\frac12\left|m(\frac m2-1)\right| \Rightarrow m(\frac m2-1)=\frac32 \Rightarrow m^2-2m-3=0 \Rightarrow (m-3)(m+1)=0$

$\boxed{\begin{matrix}m=3\\ m=-1\end{matrix}}$

$\Downarrow$

$m(\frac m2-1)=\frac{-3}{2} \Rightarrow m^2-2m+3=0 \Rightarrow$ غ ق ق

$\Delta<0$!!

$y=x^2-mx+1$

$\frac{-b}{2a} \to \frac m2 \Rightarrow \boxed{\frac32}$

$\Downarrow$

$\boxed{\frac{-1}{2}}$ یا

کمترین مقدار $\rightarrow min \rightarrow a>0$

(۶)

$ax^2+3x+a \rightarrow \frac{-b}{2a}=\frac{7}{8} \rightarrow \frac{-3}{2a}$

$x=\frac{-3}{2a}$

$\frac{9a}{4a^2}-\frac{9}{2a}+a=\frac{7}{8} \Rightarrow \frac{9-18+4a^2}{4a}=\frac{7}{8} \Rightarrow \frac{-9}{4a}+a=\frac{7}{8} \xrightarrow{\times 8a} 8a^2-18-7a=0$

نمره ۱- یک مقدار $a=2$

$a=\frac{7\pm\sqrt{49+576}}{16} \Rightarrow \frac{7\pm\sqrt{625}}{16} \Rightarrow \frac{7\pm 25}{16}$ ↗ $\boxed{a=2}$ ↘ $a=-\frac{9}{8}$ $a>0$ غ ق ق !

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$x^2-(a+1)x+a=0$ $\quad \alpha(\alpha+2)\rightarrow \alpha^2+2\alpha=a$ $\rightarrow \frac{\cancel{\alpha(\alpha+2)}}{\beta(\beta+2)} \Rightarrow 3$, $=24 \Rightarrow 24-3=\boxed{21}$

$x^2-(3a+1)x+b=0$ $\quad \beta(\beta+2)\rightarrow \beta^2+2\beta=b$

$3a+1=2\beta+2 \rightarrow \frac{3a-1}{2}=\beta \rightarrow \beta=4$, ۲۴

$a+1=2\alpha+2 \rightarrow \frac{a-1}{2}=\alpha \rightarrow \frac{a-1}{2}\times\frac{a+3}{2}=a$ $\quad$ (ریشه دومین) $\rightarrow x^2+px+q$

$\alpha=3 \rightarrow 1, 3$ $\quad (a-1)(a+3)=4a \rightarrow a^2-2a-3=0$ $\quad a=3$ ✓

$(a-3)(a+1)$ $\quad a=-1$ غ ق ق طبق ✗

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(۸)

$y=-ax^2-ax+2 \rightarrow x_y=\frac{-b}{2a}=\frac{1}{2} \rightarrow \frac{-\Delta}{4a} \rightarrow \frac{-(a^2-(4)(2)(-a))}{-4a}=\frac{a^2+8a}{4a}=\frac{a}{4}+2$

$y=2bx^2-bx-1 \rightarrow x_{y'} \rightarrow \frac{-b}{2a}=\frac{1}{4} \rightarrow 2b(\frac{1}{16})-b(\frac{1}{4})-1=\frac{a}{4}+2 \rightarrow$

$\frac{-\Delta}{4a}=\frac{-(b^2-(-1)(4)(2b))}{8b}=\frac{b^2+8b}{-8b}$ جایگذاری $\quad \frac{b}{8}-\frac{b}{4}-1=\frac{a}{4}+2 \rightarrow \frac{a}{4}=-3 \Rightarrow a=-12$

$\frac{-a}{16}+\frac{a}{4}+2=\frac{-b}{8}-1 \Rightarrow \frac{12}{16}=\frac{-b}{8} \rightarrow b=-6 \Rightarrow b-a=-6+12=\boxed{6}$

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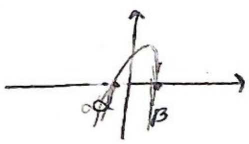
(۹)

$y=25\alpha x^2+4x+\beta \rightarrow \alpha\beta \Rightarrow \frac{\beta}{25\alpha}=\alpha\beta \rightarrow 25\alpha^2\beta=\beta$

$\beta>\alpha$ $\quad \rightarrow 5x^2+4x+1 \Rightarrow \frac{-b}{2a}=\frac{-4}{10} \rightarrow$ $\boxed{\text{ربع دوم}}$ غ ق ق $\quad 5\alpha=+1 \rightarrow \alpha=\frac{1}{5}$

$\quad \rightarrow -5x^2+4x-\frac{3}{5} \Rightarrow \frac{-b}{2a}=\frac{-4}{-10}=\boxed{\frac{4}{10}\ \text{ربع اول}}$ جواب ✓ $\quad 5\alpha=-1 \rightarrow \alpha=\frac{-1}{5}$



$\alpha+\beta \Rightarrow \frac{-4}{25\times\frac{-1}{5}}=\frac{-4}{-5}=\frac{4}{5}+\frac{1}{5}=1=\beta$, $\alpha=\frac{-1}{5}$ ق ق

$\frac{-4}{25\times\frac{1}{5}}=\frac{-4}{5}+\frac{1}{5}=\frac{-3}{5}=\beta$, $\alpha=\frac{1}{5}$ غ ق ق $\beta>\alpha$

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(۱۰)

$x^2-(a^2+b^2-12)x+a+b-1=0$

$a+b=a^2+b^2-12 \rightarrow a^2-a+b^2-b-12=0 \rightarrow$

$ab \rightarrow \frac{a+b-1}{1}=ab \rightarrow a+b-1=ab \rightarrow$

$(a+b)^2-2ab \Rightarrow (a+b)^2-2ab-12=a+b$

$(a+b)=t$ $\quad (a+b)^2-2(a+b-1)-12=a+b$

$t=t^2-2t-10 \rightarrow t^2-3t-10=0 \rightarrow (t-5)(t+2)=0$ ↗ $\boxed{a+b=5}$ ✓✓ ↘ $\boxed{a+b=-2}$ غ ق ق

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