روزنامه‌ی خدای

① الف) $y = 2x^2 - 4x + 1 \rightarrow a>0 \rightarrow \min$ ، $\left|\frac{-b}{2a} \Rightarrow \frac{-(-4)}{4}=1 \right| \Rightarrow \left|\frac{-(-8)}{8}=-1\right| \Rightarrow |1\ -1|$

$\Delta = b^2 - 4ac \Rightarrow 16 - (4)(2)(1) = 8$

ب) $y = -2x^2 + 3x - 5 \rightarrow a<0 \rightarrow \max$ ، $\left|\frac{-b}{2a} \Rightarrow \frac{-3}{-4} \Rightarrow \frac{3}{4}\right| \Rightarrow \left|\frac{-9}{-8} \Rightarrow \frac{9}{8}\right| \Rightarrow \left|\frac{3}{4}\ \frac{9}{8}\right|$

$\Delta = b^2 - 4ac \Rightarrow 9 - (4)(-2)(-5) = -31$

② الف) $y = x^2 - 4x + 1$ ، $a>0$ min

$x=0 \Rightarrow y=1 \quad (0,1)$ ext $\Rightarrow \left|\frac{-b}{2a} \Rightarrow \frac{4}{2}=2\right| \Rightarrow \left|\frac{-12}{4}=-3\right|$

$\Delta = b^2 - 4ac \Rightarrow 16 - (4)(1)(1) = 12$

$a<0$ max

$x = \frac{4 \pm \sqrt{12}}{2} \rightarrow 2 \pm \sqrt{3}$

$\rightarrow y = 4x^2 - 4x + 1$

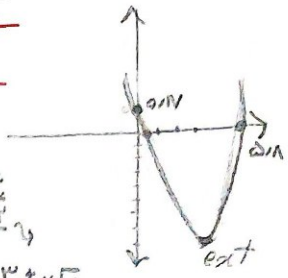
ب) $y = -x^2 + 4x + 1$

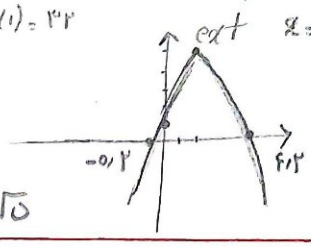
$\rightarrow x=0 \Rightarrow y=1$

ext $\Rightarrow \left|\frac{-b}{2a} \Rightarrow \frac{-4}{-2}=2\right| \Rightarrow \left|\frac{-5}{-4}\right|$ ← جایه‌داری

$\Delta = b^2 - 4ac \Rightarrow 16 - (4)(1)(-1) \Rightarrow 20$

$x = \frac{-4 \pm \sqrt{20}}{-2} \Rightarrow 2 \pm \sqrt{5}$





③ $4x^3 + 12x^2 - 9x - 2 = 0 \rightarrow$ چون ریشه سوم ندارد، تکنیک سه ریشه‌ی دیگر داریم $\Rightarrow 4(x-\beta)(x-\alpha)(x-c) \Rightarrow$

$\alpha\beta = -2$

$\alpha + \beta = 1$

$(4x^2 - 4x\alpha - 4\beta x + 4\beta\alpha)(x-c) \Rightarrow 4x^3 - 4x^2\alpha - 4x^2\beta + 4x\beta\alpha - 4cx^2 + 4x\alpha c + 4\beta c x - 4\beta\alpha c$

$\Rightarrow 4x^3 + 12x^2 - 9x - 2 = 4x^3 + x^2(-4\alpha - 4\beta - 4c) + x(4\beta\alpha + 4\alpha c + 4\beta c) - 4\beta\alpha c$

$-4(\alpha + \beta + c) = 12 \Rightarrow -4\left(1 - \frac{1}{4}\right) = 12 \rightarrow -4 \times \frac{3}{4} = -3 \neq 12$

$4(\beta\alpha + \alpha c + \beta c) = -9$

$-4\beta\alpha c = -2 \rightarrow 8c = -2 \rightarrow c = \frac{-2}{8} = \frac{-1}{4}$

④ $x^2 - 3mx + m = 0$

$\sqrt{\beta} - \sqrt{\alpha} = 1 \rightarrow (\sqrt{\beta} - \sqrt{\alpha})^2 = 1 \rightarrow \beta + \alpha - 2\beta\alpha = 1 \Rightarrow 3m - 2m = 1 \rightarrow m = 1$

$\alpha + \beta = \frac{-b}{a} = \frac{3m}{1} = 3m$

$\alpha\beta \rightarrow m$

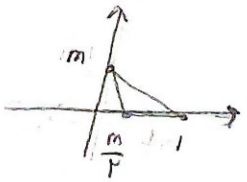
$2x^2 - mx - m = 0 \Rightarrow \alpha'\beta' = \frac{c'}{a'} \Rightarrow \frac{-m}{2} = \frac{-1}{2}$

⑤ $y = 2x^2 - (m+2)x + m \xrightarrow[x=0]{\text{تقاطع با محور}} y = m$

$\Delta = b^2 - 4ac \Rightarrow (m+2)^2 - 4(2)(m) \Rightarrow m^2 + 4 + 4m - 8m \Rightarrow m^2 + 4 - 4m \rightarrow (m-2)^2$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow \frac{m+2 \pm \sqrt{(m-2)^2}}{4}$ ↗ $m-2>0 \Rightarrow \frac{m+2+m-2}{4} = \frac{2m}{4} = \frac{1}{2}m$ ، $\frac{2m+2-m+2}{4} = 1$

↘ $m-2<0 \Rightarrow \frac{m+2-m+2}{4} = 1$ ، $\frac{m+2+m-2}{4} = \frac{1}{2}m$



$S = \frac{1}{2}\left|m\left(\frac{m}{2} - 1\right)\right| \Rightarrow m\left(\frac{m}{2} - 1\right) = \frac{3}{2} \Rightarrow m^2 - 2m - 3 = 0 \Rightarrow (m-3)(m+1) = 0$

$m = 3$
$m = -1$

$m\left(\frac{m}{2} - 1\right) = -\frac{3}{2} \Rightarrow m^2 - 2m + 3 = 0 \Rightarrow$ غ ق

$\Delta < 0$!!

$y = x^2 - mx + 1$

$\frac{-b}{2a} \rightarrow \frac{m}{2} \Rightarrow \frac{3}{2}$ یا $\frac{-1}{2}$

کمینه‌ش min → a>0

(۴)

$ax^2+2ax+a \longrightarrow \frac{-b}{2a}=\frac{7}{8} \longrightarrow =\frac{-3}{2a}$

$x=\frac{-3}{2a}$

$\frac{9a}{4a^2}-\frac{9}{2a}+a=\frac{7}{8} \Rightarrow \frac{9-18+4a^2}{4a}=\frac{7}{8} \Rightarrow \frac{-9}{4a}+a=\frac{7}{8} \xrightarrow{\times 8a} 8a^2-18-7a=0$

نقطه یک مقدار a=۲

$a=\frac{7\pm\sqrt{49+576}}{16} \xrightarrow{\Delta} \frac{7\pm\sqrt{625}}{16} \Rightarrow \frac{7\pm 25}{16}$ → $a=2$

↘ $a=\frac{-9}{8}$ a>0 قبول نیست!

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(۷)

$x^2-(a+1)x+a=0$

$x^2-(2a+1)x+b=0$

$\alpha(\alpha+2) \rightarrow \alpha^2+2\alpha=a$

$\beta(\beta+2) \rightarrow \beta^2+2\beta=b$

$\frac{3}{\alpha(\alpha+2)} \Rightarrow 3 \Rightarrow \beta(\beta+2)=24$ → $24-3=21$

$2a+1=2\beta+2 \rightarrow \frac{2a-1}{2}=\beta \longrightarrow \beta=4$ یا $\beta=-3$

$a+1=2\alpha+2 \rightarrow \frac{a-1}{2}=\alpha \longrightarrow \frac{a-1}{2}\times\frac{a+3}{2}=a$

$\alpha=2 \rightarrow 1,3$

$(a-1)(a+3)=4a \rightarrow a^2-2a-3=0$ $a=3$ ✓

$(a-3)(a+1)$ ↙ $a=-1$ غیق ضمنی ✗

(۸)

$y=-2ax^2-ax+2 \rightarrow x_y=\frac{-b}{2a}=\frac{1}{4} \rightarrow \frac{-5}{4a} \rightarrow \frac{-(a^2-(4)(2)(-a))}{-4a}=\frac{a^2+8a}{4a}=\frac{a}{4}+2$

$y=2bx^2-bx-1 \Rightarrow \frac{-D}{4a}=-\frac{(b^2-(-1)(4)(2b))}{8b}=\frac{b^2+8b}{-8b}$ ÷b → $\frac{-a}{16}+\frac{a}{4}+2=\frac{-b}{8}-1 \Rightarrow \frac{12}{16}=\frac{-b}{8} \rightarrow b=-4$

$x_{y'} \rightarrow \frac{-b}{2a}=\frac{1}{4} \rightarrow 2b\left(\frac{1}{4}\right)-b\left(\frac{1}{2}\right)-1=\frac{a}{4}+2 \rightarrow \frac{b}{2}-\frac{b}{4}-1=\frac{a}{4}+2 \rightarrow \frac{a}{4}=-4 \Rightarrow a=-12$

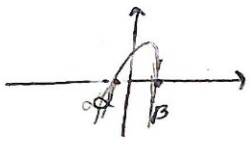
$b-a=-4+12=8$ ۴

(۹)

$y=2a\alpha x^2+4\alpha x+\beta \rightarrow \alpha\beta \Rightarrow \frac{\beta}{2a\alpha}=\alpha\beta \rightarrow 2a\alpha^2\beta=\beta$

$\beta>\alpha$ ↳ $a\alpha^2+4\alpha x+1 \Rightarrow \frac{-b}{2a}=\frac{-4}{10} \rightarrow$ مجموع ریشه‌ها غیق $a\alpha=+1 \rightarrow \alpha=\frac{1}{a}$

↳ $-a\alpha^2+4\alpha x-2 \Rightarrow \frac{-b}{2a}=\frac{-4}{-10}=\frac{4}{10}$ جواب اول ✓ $a\alpha=-1 \rightarrow \alpha=\frac{-1}{a}$



$\alpha+\beta \Rightarrow \frac{-4}{2a\times\frac{-1}{a}}=\frac{-4}{-a}=\frac{4}{a}+\frac{1}{a}=\frac{1}{a}$ $\beta=1$ $\alpha=\frac{-1}{a}$ } قبول

$\frac{-4}{2a\times\frac{1}{a}}=\frac{-4}{a}+\frac{1}{a}=\frac{-3}{a}=\beta$ $\alpha=\frac{1}{a}$ } غیق $\beta>\alpha$

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(۱۰)

$x^2-(a^2+b^2-12)x+a+b-1=0$

$a+b=a^2+b^2-12 \longrightarrow a^2-a+b^2-b-12=0$

$ab \rightarrow \frac{a+b-1}{1}=ab \rightarrow a+b-1=ab \rightarrow$

$(a+b)^2-2ab \Rightarrow (a+b)^2-2ab-12=a+b$

$(a+b)=t \quad (a+b)^2-2(a+b-1)-12=a+b$

$t=t^2-2t-10 \rightarrow t^2-3t-10=0 \rightarrow (t-5)(t+2)=0$

$a+b=5$ ✓

$a+b=-2$ غیق