پاسخ تمرینات تکلیفی ۲۴ (هم فکر)

۱)

الف) $y = 2x^2 - 4x + 1$

$$ext\left|\begin{array}{l}\frac{-b}{2a} = \frac{4}{4} = 1 \\ \frac{-\Delta}{4a} = \frac{-8}{8} = -1\end{array}\right.$$

$\Delta = b^2 - 4ac \Rightarrow 16 - 8 = 8$

ب) $y = -2x^2 + 3x - 9$

$$ext\left|\begin{array}{l}\frac{-b}{2a} = \frac{-3}{-4} = \frac{3}{4} \\ \frac{-\Delta}{4a} = \frac{31}{8}\end{array}\right.$$

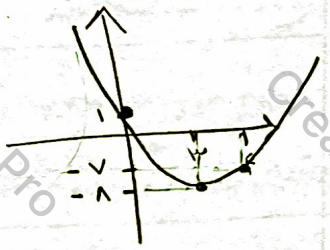
$\Delta = b^2 - 4ac \Rightarrow 9 - 72 = -31$

۲)

الف) $y = x^2 - 4x + 1$

$$ext\left|\begin{array}{l}\frac{-b}{2a} = \frac{4}{2} = 2 \\ \frac{-\Delta}{4a} = \frac{-12}{4} = -3\end{array}\right.$$

x	4	3	0	1
y	-3	-3	1	-4

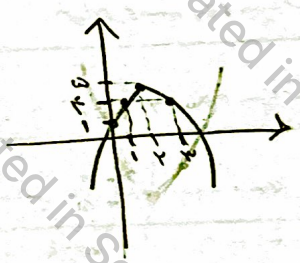


$\Delta = b^2 - 4ac \rightarrow 16 - 4 = 12$

ب) $y = -x^2 + 4x + 1$

$$ext\left|\begin{array}{l}\frac{-b}{2a} = \frac{-4}{-2} = 2 \\ \frac{-\Delta}{4a} = \frac{-20}{-4} = 5\end{array}\right.$$

x	1	2	3	0
y	4	5	4	1



$\Delta = b^2 - 4ac \Rightarrow 16 - 4(-1)(2) = 20$

۳)

$4x^3 + kx^2 - 9x - 2 = 0 \xrightarrow{x=2} a(x-\alpha)(x-\beta)(x-\gamma)$

$= ax^3 - a(\alpha+\beta+\gamma)x^2 + a(\alpha\beta+\alpha\gamma+\beta\gamma) - a\alpha\beta\gamma = 0$

$S = \frac{\alpha+\beta+\gamma}{} = \frac{-k}{4} \rightarrow 1 - \frac{1}{2} = \frac{-k}{4} \rightarrow \boxed{k = -3}$

$P = \frac{\alpha\beta\gamma}{} = \frac{+2}{4} = +\frac{1}{2} \rightarrow \alpha\beta = -2 \quad \gamma = -\frac{1}{4}$

۴)

۱) $x^2 - 2mx + m = 0 \xrightarrow{x=\alpha} \alpha^2 - 2m\alpha + m = 0$

$\xrightarrow{x=\beta} \beta^2 - 2m\beta + m = 0$

$S = 2$

$P = m$

$2x^2 - mx - m = 0$

$2x^2 - x - 1 = 0$

$P = \boxed{\frac{-1}{2}}$

$P = \frac{c}{a}$

۲) $\sqrt{\alpha} - \sqrt{\beta} = 1$ (توان ۲)

$\alpha + \beta - 2\sqrt{\alpha\beta} = 1$

$S \qquad P$

$2^2 - 2\sqrt{m} = 1 \rightarrow -2\sqrt{m} = -2 \rightarrow \sqrt{m} = 1 \rightarrow m = \pm 1 \xrightarrow{\sqrt{\ }} \boxed{m = 1}$

(۵

$y = x^2 - (m-2)x + m$

$S = \frac{1}{2} \times \frac{|m-2|}{2} \times |m| = \frac{|m(m-2)|}{4} = 3$

$|m(m-2)| = 12 \rightarrow m^2 - 2m - 12 = 0 \rightarrow (m-3)(m+1) = 0$

$m = 3 \quad m = -1$

$x_{\text{رأس}} = (0, m)$ ← مختصات رأس

$\alpha + \beta = \frac{m+2}{2}$

$\alpha\beta = \frac{m}{2}$

طول وتر $= |x_1 - x_2| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{(m-2)^2}}{2} = \frac{|m-2|}{2}$

$\Delta = (m+2)^2 - 8m \rightarrow m^2 + 4 + 4m - 8m = m^2 + 4 - 4m = (m-2)^2$

(۶

$\frac{m}{2} = \frac{3}{2}, \frac{1}{2}$

$g = ax^2 + 2x + a \qquad \Delta = 4 + 4a^2 = 9a$

$a > 0 \quad \min \quad \Delta = 9 - 4a^2 \qquad a = \frac{1 \pm 2a}{8} \rightarrow \begin{cases} a = 2 \\ a = -\frac{9}{8} \end{cases} \xrightarrow{a>0} a = 2$

ext $\left| \begin{array}{l} \frac{-b}{2a} = \frac{-2}{4a} = -\frac{1}{a} \\ \frac{-\Delta}{4a} = \frac{b^2 - 4ac}{4a} \end{array} \right. \xrightarrow{b=2} \frac{4a^2 - 8}{4a} = \frac{7}{8} \rightarrow 8(4a^2 - 8) = 7a \quad 8a^2 - 18 = 7a$

$8a^2 - 7a - 18 = 0$

(۷

$x^2 - (a+1)x + a = 0 \xrightarrow{x=\alpha} \alpha^2 - (a+1)\alpha + a = 0$

$\frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{(a-1)^2}}{|1|} = a - 1 = 2 \rightarrow a = 3$ ← طول وتر

$\xrightarrow{x=\beta} \beta^2 - (a+1)\beta + a = 0$

$S = a + 1 = 4$

$\beta_1 = a = 3$

$\Delta = b^2 - 4ac \rightarrow (a+1)^2 - 4a \rightarrow a^2 + 1 + 2a - 4a = a^2 - 2a + 1 = (a-1)^2$

$x^2 - (2a+1)x + b = 0 \xrightarrow{x=\alpha} \alpha^2 - (2a+1)\alpha + b = 0 \quad \alpha^2 - 8\alpha + b = 0$

$\xrightarrow{x=\beta} \beta^2 - (2a+1)\beta + b = 0 \quad \beta^2 - 8\beta + b = 0$

$\frac{\sqrt{\Delta}}{|a|} = \sqrt{100 - 4b} = 2$ ← تکه $\rightarrow 100 - 4b = 4$

$S = 2a + 1 = 10$

$\beta_2 = b = 24$

$\Delta = b^2 - 4ac \rightarrow (2a+1)^2 - 4b \rightarrow 100 - 4b$

$99 = 4b$

$b = \frac{99}{4} = 24$

اختلاف طول وتر بیشتر $= \beta_2 - \beta_1 = 24 - 3 = 21$

(۸

① $y = -ax^2 + ax + 2 \rightarrow$ ext $\left| \begin{array}{l} \frac{-b}{2a} = \frac{-a}{-2a} = \frac{1}{2} \\ \frac{-\Delta}{4a} = \frac{a^2 + 8a}{-4a} = \frac{-a(a+8)}{4a} = \frac{a+8}{4} \end{array} \right.$ ← طول یکی از محل‌های برخورد با محور xها

$\Delta = a^2 + 8a$

$a = -12$

② $g = 2bx^2 - bx - 1 \rightarrow$ ext $\left| \begin{array}{l} \frac{-b}{2a} = \frac{b}{4b} = \frac{1}{4} \\ \frac{-\Delta}{4a} = \frac{-(b^2 + 8b)}{8b} = \frac{-b(b+8)}{8b} = -\frac{b+8}{8} \end{array} \right.$ ← طول یکی از محل‌های برخورد با محور xها

$\Delta = b^2 + 8b$

$b = 24$

$a + b = -12 + 24 = 12$

$y = 2\omega\alpha x^2 + 4x + \beta \xrightarrow{x=\alpha} 2\omega\alpha^2 + 4\alpha + \beta = 0$

$\xrightarrow{x=\beta} 2\omega\alpha\beta^2 + 4\beta + \beta = 0 \quad 2\omega\alpha\beta^2 + \omega\beta = 0$

$\omega\beta(2\alpha\beta + 1) = 0$

$S = \frac{-4}{2\omega\alpha} = \alpha + \beta$

$P = \frac{\beta}{2\omega\alpha} = \alpha\beta \longrightarrow 2\omega\alpha^2 = 1 \longrightarrow \alpha = \pm\frac{1}{\omega}$

$\xrightarrow{\alpha < \beta} \alpha = -\frac{1}{\omega}, \beta = 1$

$y = -\omega x^2 + 4x + 1 \longrightarrow$ بعد $\left|\begin{matrix}\frac{2}{\omega} > 0 \\ \Delta > 0\end{matrix}\right. \longrightarrow$ **نامعین**

(10) $x^2 - (a^2 - b^2 - 12)x + a + b - 1 = 0 \quad S = a + b \quad P = ab$

$a^2 + b^2 - 12 = a + b \longrightarrow S^2 - 2P - 12 = S \longrightarrow S^2 - 2(S-1) - 12 = S$

$S^2 - 2S + 2 - 12 = S$

$S^2 - 3S - 10 = 0$

$ab = a + b - 1 \longrightarrow P = S - 1$

$(S-5)(S+2) = 0$

$S = 5$ ✓ $\quad S = -2$ ✗

a, b حقیقی