مونا عباسی

۱-

الف) $Min \left| \begin{array}{l} \frac{-b}{2a} = \frac{4}{4} = 1 = x_S \\ \frac{-\Delta}{4a} = 2-4+1 = -1 = y_S \end{array} \right.$

ب) $Max \left| \begin{array}{l} \frac{-b}{2a} = \frac{-3}{-4} = \frac{3}{4} = x_S \\ y_S = -4 \times \frac{9}{16} + 3 \times \frac{3}{4} - 5 = -\frac{9}{8} + \frac{18}{8} - \frac{40}{8} = -\frac{31}{8} \end{array} \right.$

۲-

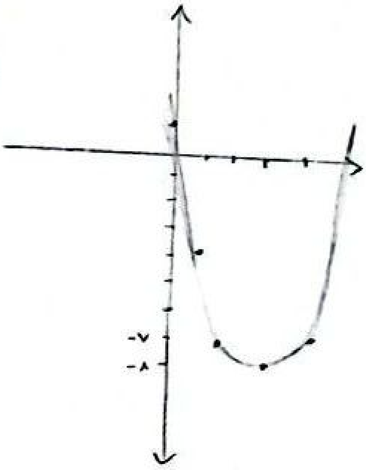
الف) $Min \left| \begin{array}{l} x_S = -\frac{b}{2a} = \frac{6}{2} = 3 \\ y_S = 9 - 18 + 1 = -8 \end{array} \right.$

x	0	3	1	4
y	1	-8	-4	-7

$y = x^2 - 6x + 1$

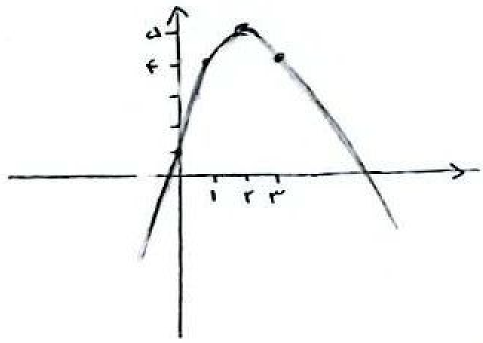
$1 - 6 + 1$

$16 - 24 + 1$



ب) $y = -x^2 + 4x + 1$ $\quad Max \left| \begin{array}{l} x_S = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y_S = -4 + 8 + 1 = 5 \end{array} \right.$

x	1	2	3
y	4	5	4



۳-

$\left. \begin{array}{l} \alpha\beta = -2 \\ \alpha + \beta = 1 \rightarrow \alpha = 1 - \beta \end{array} \right\} (1-\beta)(\beta) = -2 \longrightarrow -\beta^2 + \beta = -2 \longrightarrow \beta^2 - \beta - 2 = 0 \quad (\beta - 2)(\beta + 1) = 0 \; \begin{cases} 2 \\ -1 \end{cases}$

$4x^3 + Kx^2 - 9x - 2 = 0 \xrightarrow{x=2} 4(2)^3 + K(2)^2 - 9(2) - 2 = 0 \rightarrow 4K = -12 \; \boxed{K = -3}$

$\xrightarrow{(x=-1)} 4(-1)^3 + K(-1)^2 - 9(-1) - 2 = 0 \longrightarrow \boxed{K = -3}$

۴-

$|\sqrt{\alpha} - \sqrt{\beta}| = 1 \quad \alpha + \beta - 2\sqrt{\alpha\beta} = 1 \quad x^2 - 3mx + m = 0 \qquad S = \frac{-(-3m)}{1} = 3m \quad P = \frac{m}{1}$

$3m - 2\sqrt{m} - 1 \xrightarrow{a+b+c=0}$

$\xrightarrow{1} 1 \quad \sqrt{m} = 1 \quad m = 1$

$\xrightarrow{2} \frac{c}{a} = -\frac{1}{3} \rightarrow \sqrt{m} = -\frac{1}{3}\alpha \quad 2x^2 - mx - m = 0 \quad P = \alpha\beta = \frac{-m}{2} = \boxed{-\frac{1}{2}}$

۵-

$S = \frac{۱}{۲} \times m \times \frac{\sqrt{m^۲+۴-۴m}}{۲} = |\frac{۳}{۴}|$

① $m \geq ۲$ $\quad m^۲-۲m-۳=۰ \to \boxed{m=۳}$ ✓, $\boxed{m=-۱}$ if $m<۲$ $\Delta<۰$ غ ق ق

$m|m-۲| = |۳|$ $\quad m|m-۲|=۳$ ① $\quad m|m-۲|=-۳$ ②

② $m \leq ۲ \to -m^۲+۲m+۳=۰ \to \boxed{m=-۱}$ ✓, $\boxed{m=۳}$ if $m>۲$ $\Delta<۰$ غ ق ق

$m=۳ \to y = x^۲+۳x+۲ \to x_S = -\frac{۳}{۲}$

$m=-۱ \to y = x^۲-x+۲ \to x_S = -\frac{۱}{۲}$

۶-

کمترین مقدار $a>۰$ $\left\{ y_S = \frac{-\Delta}{۴a} = \frac{-۹-۴(a^۲)}{۴a} = \frac{۷}{۸} \Rightarrow ۳۲a^۲-۲۸a-۷۲=۰ \to ۸a^۲-۷a-۱۸=۰ \right.$

$a^۲-۷a-۱۴۴=۰ \quad (a+۹)(a-۱۶)=۰ \Rightarrow$ ریشه‌ها $\left\{ \begin{array}{l} ① -\frac{۹}{۸} \times \quad a>۰ \\ ② \frac{۱۶}{۸}=۲ ✓ \end{array} \right.$ $\boxed{D=\{۲\}}$

۷-

$x^۲-(a+۱)x+a=۰ \quad \frac{\sqrt{\Delta}}{|a|}=۲ \quad \sqrt{\underbrace{a^۲+۱+۲a-۴a}_{(a-۱)^۲}}=۲ \quad |a-۱|=۲$; $a>۱$: $a=۳$; $a<۱$: $a=-۱$

$\to a>۱ \quad a=۳ \quad x^۲-۴x+۳=۰ \quad (x-۱)(x-۳)=۰$ ✓ $\quad P_۱ = \frac{۳}{۱}$

$a<۱ \quad a=-۱ \quad x^۲-۱=۰ \quad x=\pm ۱$ عدد طبیعی نیست

$x^۲-(۳a+۱)x+b=۰ \to \frac{\sqrt{\Delta}}{|a|}=۲$

$\sqrt{۹a^۲+۱+۶a-۴b}=۲ \xrightarrow{a=۳} \sqrt{۸۱+۱+۱۸-۴b}=۲ \to ۱۰۰-۴b=۴ \quad b=۲۴$

$|P_۲-P_۱| = |۲۴-۳| = \boxed{۲۱}$

$P_۲ = \frac{۲۴}{۱} = ۲۴$

۸-

① $y=-ax^۲+ax+۲ \to ext \left| \begin{array}{l} \frac{-a}{۲(-a)} = \frac{۱}{۲} \\ -\frac{a}{۴}+\frac{a}{۲}+۲ = \frac{a+۸}{۴} \end{array} \right.$

② $y=۲bx^۲-bx-۱ \to ext \left| \begin{array}{l} \frac{-(-b)}{۲(۲b)} = \frac{۱}{۴} \\ \frac{b}{۸}-\frac{b}{۴}-۱ = \frac{-b-۸}{۸} \end{array} \right.$

② $x=\frac{۱}{۲} \quad y=\frac{a+۸}{۴} \to \frac{b}{۲}-\frac{b}{۲}-۱ = \frac{a+۸}{۴} \quad a=-۱۲$

① $x=\frac{۱}{۴} \quad y=\frac{-b-۸}{۸}$

$\frac{-a}{۱۶}+\frac{a}{۴}+۲ = \frac{-b-۸}{۸}$

$\frac{۱۲-۴۸+۳۲}{۱۶} = \frac{-۲b-۱۶}{۱۶}$

$-۴ = -۲b-۱۶ \quad b=-۶ \quad b-a = -۶-(-۱۲) = \boxed{۶}$

$y = 25\alpha x^2 + 4x + \beta \Rightarrow \begin{cases} ① \; 25\alpha^3 + 4\alpha + \beta = 0 \to 25\alpha^3 + 4\alpha - 5\alpha = 0 \quad \alpha(25\alpha^2 - 1) = 0 \\ ② \; 50\beta^2\alpha + 5\beta = 0 \end{cases}$

$\alpha(5\alpha - 1)(5\alpha + 1) = 0$

$\begin{cases} ① \; \alpha = 0 \quad \text{غق} \\ \alpha = \frac{1}{5} \quad \beta = -1 \quad \beta < \alpha \quad \text{غق} \\ \alpha = -\frac{1}{5} \quad \beta = 1 \; ✓ \end{cases}$

$5\beta(5\alpha\beta + 1) = 0 \to ① \; \beta = 0 \quad \delta = \frac{-4}{25\alpha}$

$= 0 + \alpha \quad 25\alpha^2 = 4$ غق

$②\; 5\alpha\beta + 1 = 0 \to \alpha\beta = -\frac{1}{5} = \frac{\beta}{25\alpha} \to \beta = -5\alpha$

$\alpha = -\frac{1}{5} \quad \beta = 1 \quad -5x^2 + 4x + 1 \Rightarrow ext \left| \begin{array}{l} \frac{-4}{2(-5)} = \frac{2}{5} \\ -5\left(\frac{2}{5}\right)^2 + 4\left(\frac{2}{5}\right) + 1 = \frac{9}{5} \end{array} \right.$ $\boxed{\text{ناحیه ۱}}$

$a + b = S \quad ab = P \quad S = -(-(a^2 + b^2 - 12)) = S^2 - 2P - 12$ ۱۰

$P = a + b - 1 = S - 1$

$S^2 - 2(S - 1) - 12 = S \quad S^2 - 2S - 10 = S \quad S^2 - 3S - 10 = 0 . \quad (S - 5)(S + 2) = 0$

$S \begin{cases} 5 = a + b \quad P = 5 - 1 = 4 \; ✓ \\ -2 = a + b \quad P = a + b - 1 = -2 - 1 = -3 \end{cases}$

چون $P < 0$ یعنی یکی از ریشه ها منفی می‌شود و چون باید طبیعی باشد غیر قابل قبول ✗