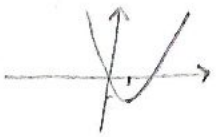
۱) الف) $y = 2x^2 - 2x$

$a > 0 \rightarrow \min$ $ext \rightarrow \left| \frac{-b}{2a} \rightarrow \frac{2}{4} = \frac{1}{2} \right.$

$\frac{-\Delta}{4a} \rightarrow \frac{-b^2}{4a} = \frac{-4}{8} = -\frac{1}{2}$



نمودار سهمی

ب) $y = -x^2 + 4x$

$a < 0 \rightarrow \max$

$\left| \frac{-b}{2a} \rightarrow \frac{-4}{-2} = 2 \right.$

$\frac{-\Delta}{4a} \rightarrow \frac{-b^2}{4a} = \frac{-16}{-4} = 4$

نمودار سهمی

۲) الف) $y = 2x^2 - 5x + 2$

$a > 0$ min $ext \left| \frac{-b}{2a} \Rightarrow \frac{5}{4} \right.$

$\frac{-\Delta}{4a} \Rightarrow \frac{-(25-16)}{8} = \frac{-9}{8}$

نمودار اول و دوم و چهارم

ب) $y = -x^2 + 4x - 1$

$a < 0$ max $ext \left| \frac{-b}{2a} \Rightarrow \frac{-4}{-2} = 2 \right.$

$\frac{-\Delta}{4a} \Rightarrow \frac{-(16-4)}{-4} = 3$

نمودار اول و سوم و چهارم

$x^2 - x - 3 = 0$

۳) الف) $\frac{\alpha+\beta}{\alpha-\beta} = \frac{\frac{-b}{a}}{\frac{\sqrt{\Delta}}{|a|}} = \frac{\frac{1}{1}}{\frac{\sqrt{1-(-3)(4)}}{1}} = \frac{1}{\sqrt{13}}$ $\quad S = -\frac{b}{a} = \frac{1}{1} = 1$, $P = -3$

ب) $\alpha^2 + \beta^2 \rightarrow S^2 - 2P = 1 - 2(-3) \Rightarrow 7$

ج) $\alpha^3 + \beta^3 \rightarrow S^3 - 3SP \Rightarrow 1 - 3(1)(-3) = 10$

د) $\alpha^3 - \beta^3 \rightarrow (\alpha-\beta)^3 + 3\alpha\beta(\alpha-\beta) \rightarrow (\sqrt{13})^3 + 3 \times -3(\sqrt{13}) \rightarrow 13\sqrt{13} - 9\sqrt{13} = 4\sqrt{13}$

$\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} \Rightarrow \sqrt{13}$

۴) $y = (x-2)(x^2 - ax + a)$

$\Delta = 0$ $a^2 - 4a = 0 \rightarrow a(a-4) = 0$ $\quad \frac{0 \quad 4}{+ \quad - \quad +} \rightarrow a \neq 0, a \neq 4$

$\Delta < 0$ $a^2 - 4a < 0 \rightarrow a(a-4) < 0$ $\quad \frac{0 \quad 4}{+ \quad - \quad +}$ $(0, 4)$

$\rightarrow (0, 4) - a$

$3x^2 - 12x - a = 0 \rightarrow 3x^2 - 12x = a$

$2\alpha^2 + \beta^2 - 4\alpha = 7$

$3\alpha^2 - 12\alpha = a \rightarrow \alpha^2 - 4\alpha = \frac{a}{3}$

$\alpha^2 + \beta^2 + \frac{a}{3} = 7 \rightarrow 14 + \frac{2a}{3} + \frac{a}{3} = 7 \rightarrow 14 + a = 7 \rightarrow a = -9$

$\frac{a}{\alpha} = \frac{-9}{+3} = -3$ ب) $\leftarrow \alpha^2 + \beta^2 = S^2 - 2P \Rightarrow 4 - 2\left(\frac{-9}{3}\right) = 10 \Rightarrow 14 + \frac{2a}{3}$

$3\alpha^2 - 12\alpha + 9 = 0 \rightarrow \alpha = \frac{+12 \pm 6}{6} \rightarrow \frac{9}{3} = 3, \frac{3}{3} = 1$

$\alpha = \frac{+9}{3} = 3$

A | $2a + 2 \rightarrow 2$, $a - 2 \rightarrow -2, 5$

B | $7 - 2a$ و نامعادله, $a - 2 \rightarrow -2, 5 \rightarrow \frac{2A + 3B}{2} = 6$

$7 + 2a + 3a - 6 = b$

S | $\frac{b}{b-2} \Rightarrow \left| \frac{5}{3} \right.$

$\rightarrow \frac{-b}{2a} = 5 \rightarrow \begin{cases} 25a + 5b + c = 3 \\ a + b + c = 1 \end{cases} \rightarrow$

$25a - 50a + c = 3$

$a - 10a + c = 1$ $\Rightarrow -\left| \begin{matrix} -25a + c = 3 \\ -9a + c = 1 \end{matrix} \right.$

$-16a = 2 \rightarrow a = -\frac{1}{8}$

$1 - \frac{9}{8} = c = \frac{-1}{8}$

$y = ax^2 + bx + c$

$a = 3 \rightarrow (1, 1)$

$(9, 1)$

$6a = 3$

$\frac{1}{8}$

۷)

$ax^2 - ax - b = 0$

$\alpha+\beta \to \frac{+a}{a} = 1 \to a\beta^2 - a\beta = b$

$\alpha\beta \to \frac{-b}{a} = \frac{1}{20} \to a(\beta^2-\beta) = -b \to \beta^2-\beta = \frac{b}{a}$

$20\beta^2 + 20\alpha^2 - 20\beta = 17$

$\div 20 \to 2\beta^2 + \alpha^2 - \beta = \frac{17}{20} \to \frac{b}{a} + \beta^2 + \alpha^2 = \frac{17}{20} \to \frac{3b}{a} = \frac{17}{20} - 1 = \frac{-3}{20} \to \frac{b}{a} = \frac{-1}{20}$

$a = -20b$

$\delta^2 : 2p \to 1 - 2\frac{-b}{a} = 1 + \frac{2b}{a}$

$\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} \Rightarrow \frac{\sqrt{a^2+4ba}}{a} \Rightarrow \sqrt{\frac{400b^2 + 80b^2}{400b^2}} \Rightarrow \frac{2}{\sqrt{5}}$

۸)

$\left|\begin{matrix}-5 & 1\\ \beta & \beta\end{matrix}\right| \to \frac{-5+1}{2} = x_S \to \left|\begin{matrix}-2 & \\ \frac{-1}{2} & \end{matrix}\right| = \frac{-b}{2a} = -2 \to -b = -4a$

$b = 4a \to \boxed{b = 2}$

$y = ax^2 + bx + c \to -\frac{1}{2} = ax^2 + 4ax + \frac{3}{2} \Rightarrow \frac{1}{2} = 4a + \cancel{4ax} - \cancel{2} + \frac{3}{2} \Rightarrow \frac{-1}{2} = -4a + \frac{3}{2}$

$-8a$

$a = \frac{1}{2}$

$y = \frac{1}{2}x^2 + 2x + \frac{3}{2} \xrightarrow{x=1} y = \frac{1}{2} + 2 + \frac{3}{2} \to \boxed{\beta = 4}$ جواب

۹)

$x^2 + 4x + a = 0 \to x_S \Rightarrow \frac{-b}{2a} \Rightarrow \frac{-4}{2} = -2$

$a = \alpha\beta \to (-2-2\sqrt{2})(-2+2\sqrt{2}) \Rightarrow 9 - 8 = \boxed{1}$

$\alpha < \beta < 0$

$-2-2\sqrt{2}$

$-2+2\sqrt{2}$

$\alpha = x_S - 2$

$\beta = x_S + 2$ — نادرست

$2\alpha^2 + 2\beta^2 = 12\sqrt{2} + 18$

$2(-2-2)^2 + 2(-2+2)^2 = 12\sqrt{2} + 18$

$32 + \cancel{32\sqrt{2}} + 18 + 18 + \cancel{32\sqrt{2}} - 12\sqrt{2} - 18 = 0$

$\leftarrow 32\sqrt{2} + 32 - 12\sqrt{2} - 50 = 0$

$\to$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} \to \frac{-4 \pm \sqrt{32 - (4)(-12\sqrt{2} - 50)(5)}}{10}$

$= \frac{-4 \pm \sqrt{32 + 240\sqrt{2} + 800}}{10}$ — نادرست

$\sqrt{32 + 240\sqrt{2} + 800} = 12\sqrt{2}$ — نادرست

۱۰)

$3x^2 - (m+14)x + 1 = 0$

$\to S = \frac{m+14}{32} \quad \alpha+\beta$

$\to P = \frac{1}{32} \quad \alpha\beta$

$\sqrt{\frac{1}{\beta}} + \sqrt{\frac{1}{\alpha}} = 5$ — به توان ۲

$\frac{1}{\beta} + \frac{1}{\alpha} + 2\sqrt{\frac{1}{\alpha\beta}} = 25$

$\frac{\alpha+\beta}{\alpha\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 25 \to \frac{\frac{m+14}{32}}{\frac{1}{32}} + 2\sqrt{32} = 25$

$m+14 = 12$

$m = -1$

$mx^2 + 2x + 2 \xrightarrow{m=-1} -x^2 + 3x + 2 \to P = \frac{c}{a} = \frac{2}{-1} = -2$