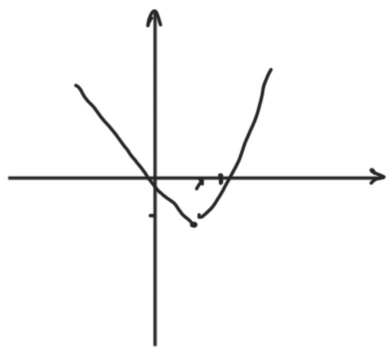
پرنیان دهقانی

۲۰

-۱

الف) $y = 3x^2 - 2x$



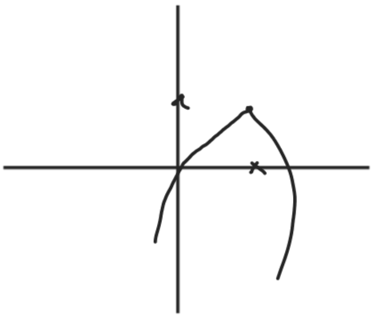
$ext \left| \begin{array}{l} \frac{2}{3} \\ \frac{-4}{12} = -\frac{1}{3} \end{array} \right.$

از ربع سوم

۳

از ربع دوم

ب) $-x^2+4x$



$ext \left| \begin{array}{l} 4 \\ \frac{-16}{-4} = 4 \end{array} \right.$

-۲ از ربع اول و دوم رسم

الف

$ext = \left| \begin{array}{l} 2/5 \\ -\frac{25-16}{8} = -\frac{9}{8} \end{array} \right.$

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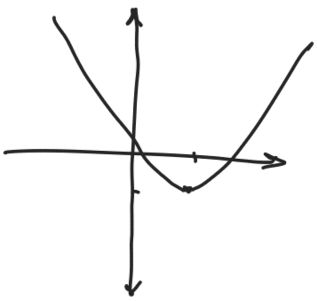
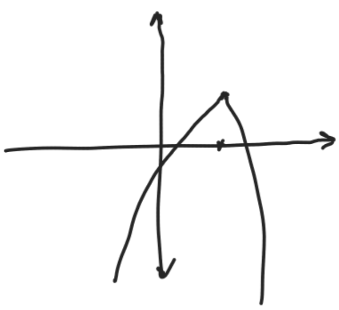


از ربع اول و سوم و چهارم

ب) $-x^2+4x-1$



$ext \left| \begin{array}{l} 2 \\ -\frac{\overset{12}{\cancel{16-4}}}{-4} = 3 \end{array} \right.$

-۳ الف)

۲

$x^2-x-3 \Rightarrow \Delta = 13$

$\alpha+\beta = \frac{1}{2}$

$\frac{\sqrt{13}}{26} = \frac{1}{2\sqrt{13}} = \frac{\frac{1}{2}}{\sqrt{13}} \leftarrow$

$\alpha-\beta = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{13}}{1} = \sqrt{13}$

$\alpha\beta = -3$

ب)

$\alpha^2+\beta^2 = S^2-2P \rightarrow \frac{1}{4} - 2\sqrt{13}$

ج) $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$
$=1^3-3(-3)(1)=1+9=10$

د) $\alpha^3-\beta^3=(\alpha-\beta)^3+3\alpha^2\beta-3\alpha\beta^2$
$=(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)$
$=(\sqrt{13})^3+3(-3)(\sqrt{13})=13\sqrt{13}-9\sqrt{13}=4\sqrt{13}$

۴ـ $x^2-ax+a \longrightarrow$ ریشه ندارد $\longrightarrow a^2-4a<0 \qquad a(a-4)<0$

۲

	$-\infty$	0	4	$+\infty$
a	$-$	$+$	$+$	
$a-4$	$-$	$-$	$+$	
$a(a-4)$	$+$	$-$	$+$	

برای بازه $(0,4)$

۵ـ $3x^2-12x-a=0 \qquad \alpha+\beta=4$

$\frac{-(-12)}{3}=\frac{12}{3}=4 \qquad -\frac{a}{3}$

$2\alpha^2+\beta^2-4\alpha=7 \to \alpha^2+\beta^2+\alpha^2-4\alpha=7$

$\alpha^2+\beta^2 \Rightarrow (\alpha+\beta)^2-2\alpha\beta=(4)^2-2(-\frac{a}{3})$
$=16+\frac{2a}{3}$

$3\alpha^2-12\alpha-a=0 \xrightarrow{\div 3} \alpha^2-4\alpha=\frac{a}{3}$

$16+\frac{2}{3}a+\frac{a}{3}=7 \to 16+a=7$

$\boxed{a=-9} \qquad \frac{-9}{3}=-3$

$3x^2-12x+9 \Rightarrow (x^2-4x+3) \to (x-1)(x-3)$ ریشه بزرگتر $=3$

۳

۶ـ $7-2a+2a+3 \Rightarrow \frac{10}{2}=5 \to (5)\ (a,3)$

$7-2a\geq 1 \to 6\geq 2a \to 3\geq a$

$2a+3=5 \to a=1$

$a-2\geq 1 \qquad a\geq 3$

$2a+3\geq 1 \qquad a\geq -1 \to$ نقاط $(1,1),(9,1)$

۴

$y=a(x-5)^2+3 \xrightarrow{(1,1)} 1=a(1-5)^2+3$

$1=a(-4)^2+3 \to 16a+3=1 \to 16a=-2 \to a=-\frac{1}{8}$

فاصله از عرض $=|-\frac{1}{8}|=\frac{1}{8}$

۷ـ $ax^2-ax-b=0 \qquad \alpha+\beta=\frac{-a}{-a}=1 \qquad \alpha\beta=\frac{-b}{a}$

۴

$40\beta^2+20\alpha^2+20\alpha=17 \to 2\beta^2+\alpha^2+\alpha=\frac{17}{20} \to \alpha^2+\beta^2=(\alpha+\beta)-\alpha\beta=1+\frac{b}{a}$

$a\beta^2-a\beta=b \to \beta^2-\beta=\frac{b}{a} \qquad 1+2\frac{b}{a}+\frac{b}{a}=1+\frac{3b}{a}=\frac{17}{20} \to \frac{b}{a}=-\frac{1}{20} \quad b=-\frac{a}{20}$

$b^2-4ac=a^2-4a(-b)=a^2+4ab=a^2+4a(\frac{-a}{20})=a^2-\frac{1}{5}a^2=\frac{4}{5}a^2$

$$\frac{\sqrt{\frac{4}{5}a^2}}{|a|} = \frac{\frac{2|a|}{\sqrt5}}{|a|} = \frac{2}{\sqrt5} = \frac{2\sqrt5}{5} \to \frac{2}{\sqrt5} = \frac{2\sqrt5}{5}$$

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۸ -

$-5+1=-4 \xrightarrow{\div 2} -2 \to$ طول راس

عرض راس $= -\frac{1}{2}$

$-\frac{b}{2a} = -2 \to -4a=-b$

$b=4a$

$y=ax^2+bx+\frac{3}{2} \to y=ax^2+4ax+\frac{3}{2}$

$\boxed{4\times\frac{1}{2}=2}$

$$\frac{4ac-b^2}{4a} = \frac{\cancel{4\times a}\times\frac{3}{2} - \cancel{16a^2}^{\,4a}}{\cancel{4a}} = \frac{3}{2}-4a = -\frac{1}{2} \to -4a=-2 \to a=\frac{1}{2}$$

$$y=\frac{1}{2}x^2+2x+\frac{3}{2} \to \beta = \frac{1}{2}\times1^2+2\times1+\frac{3}{2} = \frac{1}{2}+2+\frac{3}{2} = \frac{4}{2}+2$$

$\boxed{\beta=4}$

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۹ -

$$x^2+6x+a=0 \xrightarrow{\div 2} \frac{x^2}{2}+3x+\frac{a}{2}=0$$

$$\Delta = b^2-4ac = 3^2-4\left(\frac{1}{2}\right)\left(\frac{a}{2}\right) = 9-a$$

$$x = \frac{-3\pm\sqrt{9-a}}{2\left(\frac{1}{2}\right)} = 3\pm\sqrt{9-a}$$

$\alpha = 3-\sqrt{9-a}$ $\qquad$ $\beta = 3+\sqrt{9-a}$

$$3\alpha^2+2\beta^2 = 3\left(-3+\sqrt{9-a}\right)^2+2\left(3+\sqrt{9-a}\right)^2$$

$$= 3\left(9+9-a+6\sqrt{9-a}\right)+2\left(9+9-a-6\sqrt{9-a}\right) = 12\sqrt{2}+85$$

$$90-5a+6\sqrt{9-a} = 12\sqrt{2}+85 \to 5-5a+6\sqrt{9-a} = 12\sqrt{2}$$

اگر $a=1$ باشد
معادله برقرار

حل ۱

$$\frac{1}{\sqrt{\alpha}}+\frac{1}{\sqrt{\beta}}=\frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}}=5 \rightarrow 5\sqrt{\alpha\beta}=\sqrt{\alpha}+\sqrt{\beta}$$

توان ۲

$$(5\sqrt{\alpha\beta})^2=(\sqrt{\alpha}+\sqrt{\beta})^2$$

$$25\alpha\beta=\alpha+\beta+2\sqrt{\alpha\beta}$$

$$36x^2-(m+14)x+1=0$$

$$\text{ضرب ریشه‌ها}=\frac{1}{36} \rightarrow \frac{m+14}{36}=\text{جمع ریشه‌ها}$$

$$\frac{25}{36}=\frac{m+14}{36}+2\sqrt{\frac{1}{36}} \hookrightarrow \frac{2}{6}$$

$$\frac{25}{36}=\frac{m+14}{36}+\frac{12}{36} \rightarrow m=-1$$

$$-x^2+3x+2 \rightarrow \text{ضرب ریشه‌ها}=\frac{c}{a}=\boxed{-2}$$