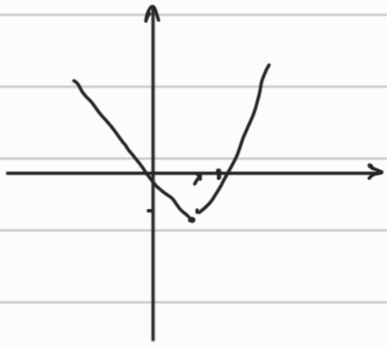
پرنیان دهقانی

۱- الف) $y = 3x^2 - 2x$

$ext \left| \begin{array}{l} \frac{2}{3} \\ \frac{-4}{12} = -\frac{1}{3} \end{array} \right.$

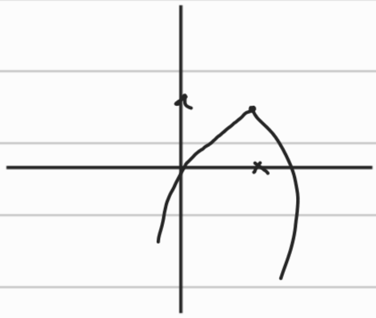
از ربع سوم



ب) $-x^2 + 4x$

از ربع دوم

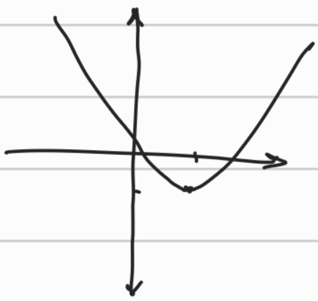
$ext \left| \begin{array}{l} 4 \\ \frac{-16}{-4} = 4 \end{array} \right.$



۲- از ربع اول دوم سوم

الف

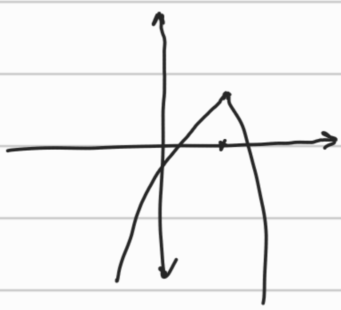
$ext = \left| \begin{array}{l} 2.5 \\ -\frac{25-16}{8} = -\frac{9}{8} \end{array} \right.$



از ربع اول و سوم و چهارم

ب) $-x^2 + 4x - 1$

$ext \left| \begin{array}{l} 2 \\ -\frac{\cancel{16-4}\ 12}{-4} = 3 \end{array} \right.$



۳- الف)

$x^2 - x - 3 \Rightarrow \Delta = 13$

$\alpha + \beta = \frac{1}{2}$

$\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{13}}{1} = \sqrt{13}$

$\frac{\sqrt{13}}{26} = \frac{1}{2\sqrt{13}} = \frac{\frac{1}{2}}{\sqrt{13}} \leftarrow$

$\alpha\beta = -3$

ب)

$\alpha^2 + \beta^2 = S^2 - 2P \longrightarrow \frac{1}{4} - 2\sqrt{13}$

ج) $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$=1^3-3(-3)(1)=1+9=10$

د) $\alpha^3-\beta^3=(\alpha-\beta)^3+3\alpha^2\beta-3\alpha\beta^2$

$=(\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)$

$=(\sqrt{13})^3+3(-3)(\sqrt{13})=13\sqrt{13}-9\sqrt{13}=4\sqrt{13}$

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۴) $x^2-ax+a \longrightarrow$ ریشه ندارد $\longrightarrow a^2-4a<0 \qquad a(a-4)<0$

	$-\infty$	0	4 $+\infty$
a	$-$	$+$	$+$
$a-4$	$-$	$-$	$+$
$a(a-4)$	$+$	$-$	$+$

برای بازه $(0,4)$

۵_ $3x^2-12x-a=0 \qquad \alpha+\beta=4$

$\frac{-(-12)}{3}=\frac{12}{3}=4 \qquad -\frac{a}{3}$

$2\alpha^2+\beta^2-4\alpha=7 \to \alpha^2+\beta^2+\alpha^2-4\alpha=7$

$\alpha^2+\beta^2 \Rightarrow (\alpha+\beta)^2-2\alpha\beta=(4)^2-2(-\frac{a}{3})$

$=16+\frac{2a}{3}$

$3\alpha^2-12\alpha-a=0 \xrightarrow{\div 3} \alpha^2-4\alpha=\frac{a}{3}$

$16+\frac{2}{3}a+\frac{a}{3}=7 \to 16+a=7 \qquad \boxed{a=-9} \qquad \frac{-9}{3}=-3$

$3x^2-12x+9 \Rightarrow (x^2-4x+3) \to (x-1)(x-3)$ ریشه بزرگتر $=3$

۶_ $7-2a+2a+3 \Rightarrow \frac{10}{2}=5 \to (5)\ (a,3)$ $\qquad 7-2a\geq 1 \to 6\geq 2a \to 3\geq a$

$2a+3=5 \to a=1$

$a-2\geq 1 \qquad a\geq 3 \qquad 2a+3\geq 1 \qquad a\geq -1 \to$ اشکال $(1,1),(9,1)$

$y=a(x-5)^2+3 \xrightarrow{(1,1)} 1=a(1-5)^2+3$

$1=a(-4)^2+3 \to 16a+3=1 \to 16a=-2 \to a=-\frac{1}{8}$

فاصله از عرض $=|-\frac{1}{8}|=\frac{1}{8}$

۷_ $ax^2-ax-b=0 \qquad \alpha+\beta=\frac{-a}{-a}=1 \qquad \alpha\beta=\frac{-b}{a}$

$40\beta^2+20\alpha^2+20\alpha=17 \to 2\beta^2+\alpha^2+\alpha=\frac{17}{20} \to \alpha^2+\beta^2=(\alpha+\beta)-\alpha\beta=1+\frac{b}{a}$

$a\beta^2-a\beta=b \to \beta^2-\beta=\frac{b}{a} \qquad 1+\frac{2b}{a}+\frac{b}{a}=1+\frac{3b}{a}=\frac{17}{20} \to \frac{b}{a}=-\frac{1}{20} \qquad b=-\frac{a}{20}$

$b^2-4ac=a^2-4a(-b)=a^2+4ab=a^2+4a(\frac{-a}{20})=a^2-\frac{1}{5}a^2=\frac{4}{5}a^2$

$$\frac{\sqrt{\frac{4}{5}a^2}}{|a|} = \frac{\frac{2|a|}{\sqrt{5}}}{|a|} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} \rightarrow \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

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۸ -

$-5+1=-4 \xrightarrow{\div 2} -2 \rightarrow$ طول راس   عرض راس $= -\frac{1}{2}$ $-\frac{b}{2a} = -2 \rightarrow -4a = -b$, $b = 4a$

$$y = ax^2 + bx + \frac{3}{2} \rightarrow y = ax^2 + 4ax + \frac{3}{2}$$

$$\boxed{4 \times \frac{1}{2} = 2}$$

$$\frac{4ac-b^2}{4a} = \frac{\cancel{4}\times a \times \frac{3}{2} - 16a^2}{\cancel{4a}} = \frac{3}{2} - 4a = -\frac{1}{2} \rightarrow -4a = -2 \rightarrow a = \frac{1}{2}$$

$$y = \frac{1}{2}x^2 + 2x + \frac{3}{2} \rightarrow \beta = \frac{1}{2}\times 1^2 + 2\times 1 + \frac{3}{2} = \frac{1}{2} + 2 + \frac{3}{2} = \frac{4}{2} + 2$$

$$\boxed{\beta = 4}$$

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۹ -

$$x^2 + 6x + a = 0 \xrightarrow{\div 2} \frac{x^2}{2} + 3x + \frac{a}{2} = 0$$

$$\Delta = b^2 - 4ac = 3^2 - 4\left(\frac{1}{2}\right)\left(\frac{a}{2}\right) = 9 - a \qquad x = \frac{-3 \pm \sqrt{9-a}}{2\left(\frac{1}{2}\right)} = 3 \pm \sqrt{9-a}$$

$$\alpha = 3 - \sqrt{9-a} \qquad \beta = 3 + \sqrt{9-a}$$

$$3\alpha^2 + 2\beta^2 = 3\left(-3+\sqrt{9-a}\right)^2 + 2\left(3+\sqrt{9-a}\right)^2$$

$$= 3\left(9+9-a+6\sqrt{9-a}\right) + 2\left(9+9-a-6\sqrt{9-a}\right) = 12\sqrt{2} + 85$$

$$90 - 5a + 6\sqrt{9-a} = 12\sqrt{2} + 85 \rightarrow 5 - 5a + 6\sqrt{9-a} = 12\sqrt{2}$$

اگر $a = 1$ باشد
معادله برقرار

۱۰

$$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}} = 5 \rightarrow 5\sqrt{\alpha\beta} = \sqrt{\alpha}+\sqrt{\beta}$$

توان ۲

$$(5\sqrt{\alpha\beta})^2 = (\sqrt{\alpha}+\sqrt{\beta})^2$$

$$25\alpha\beta = \alpha+\beta+2\sqrt{\alpha\beta}$$

$$36x^2 - (m+14)x + 1 = 0$$

ضرب ریشه‌ها $= \frac{1}{36} \rightarrow \frac{m+14}{36} =$ جمع ریشه‌ها

$$\frac{25}{36} = \frac{m+14}{36} + 2\sqrt{\frac{1}{36}}$$

$$\hookrightarrow \frac{2}{6}$$

$$\frac{25}{36} = \frac{m+14}{36} + \frac{12}{36} \rightarrow m = -1$$

$-x^2 + 3x + 2 \rightarrow$ ضرب ریشه‌ها $= \frac{c}{a} = \boxed{-2}$