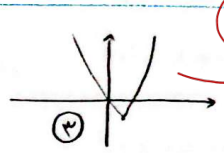
۲۵

سوال ۱)

الف) $Min \left| \begin{array}{l} -\frac{b}{2a} = \frac{2}{4} = \frac{1}{2} \\ 2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) = -\frac{1}{2} \end{array} \right.$ نقطه می دهیم: $x=0 \rightarrow y=0$



ناحیه سوم

ب) $Max \left| \begin{array}{l} -\frac{b}{2a} = \frac{-4}{-2} = 2 \\ -2^2 + 4(2) = 4 \end{array} \right.$ نقطه می دهیم: $x=0 \rightarrow y=0$

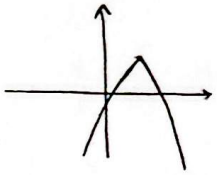
ناحیه دوم

سوال ۲)

الف) $Min \left| \begin{array}{l} -\frac{b}{2a} = \frac{5}{4} \\ -\frac{\Delta}{4a} = \frac{-b^2+4ac}{4a} = \frac{-25+4\times 2\times 2}{4\times 2} = \frac{-9}{8} \end{array} \right.$ نقطه می دهیم: $x=0$, $y=2$

۱ و ۲ و ۴

ب) $Max \left| \begin{array}{l} \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ -4+8-1 = 3 \end{array} \right.$ نقطه می دهیم: $x=0 \rightarrow y=-1$



۱ و ۳ و ۴

سوال ۳)

$|\alpha-\beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{b^2-4ac}}{|a|} = \sqrt{(-1)^2-4(1)(-3)} = \sqrt{13}$

P (ضرب) $= \frac{c}{a} = -3$

S (جمع) $= \frac{-b}{a} = \frac{-(-1)}{1} = 1$

الف) $\frac{\alpha+\beta}{\alpha-\beta} = \frac{1}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

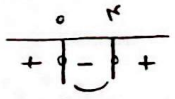
ب) $(\alpha+\beta)^2 - 2\alpha\beta = S^2 - 2P = 1 - 2(-3) = 7$

ج) $(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta) = (\alpha+\beta)((\alpha-\beta)^2+\alpha\beta)$
$= 1 \times ((\sqrt{13})^2 + (-3)) = 10$

د) $(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta) = (\alpha-\beta)((\alpha+\beta)^2-\alpha\beta)$
$= (\sqrt{13})(1^2-(-3)) = 4\sqrt{13}$

سوال ۴)

① $\Delta < 0 \rightarrow b^2-4ac = a^2-4a < 0$
$a(a-4) < 0$



$(0,4)$ $\Longrightarrow (0,4]$

② $\Delta = 0$, $(x-2)^2 = x^2-4x+4 \rightarrow a = 4$

سوال ۵)

$S = \frac{-b}{a} = \frac{12}{3} = 4$ $P = \frac{c}{a} = \frac{-a}{3}$ $3\alpha^2-12\alpha-a=0$ $3\beta^2-12\beta-a=0$

$3(\alpha^2-4\alpha) = a \rightarrow \alpha^2-4\alpha = \frac{a}{3}$

$(\alpha^2+\beta)^2+(\alpha^2-4\alpha) = 7 \rightarrow (\alpha+\beta)^2-2\alpha\beta+(\alpha^2-4\alpha) = 7$

$\rightarrow 4^2-2\left(-\frac{a}{3}\right)+\frac{a}{3} = 7 \Longrightarrow 16+a = 7 \rightarrow a = -9$

$3x^2-12x+9 = 0$

$a+b+c = 0 \rightarrow x = 1$

$\frac{c}{a} = \frac{9}{3} = 3$ ریشه ی بزرگتر

$\frac{-9}{3} = -3$ برابر

سوال ۶)

y ها یکسان → محور تقارن سهمی : $\frac{7-2a+2a+3}{2} = 5$ → طول رأس سهمی

$S \left| \begin{matrix} b=5 \\ b-2 = 5-2 = 3 \end{matrix} \right.$

$y = ax^2+bx+c \rightarrow \frac{-b}{2a} = 5 \rightarrow b = -10a$

$\left| \begin{matrix} 5 \\ 3 \end{matrix} \right. \rightarrow 3 = 25a - 50a + c$

$c = 25a+3$

$y = ax^2 - 10ax + 25a + 3$

طول های B , A طبیعی هستند ⇒

$\begin{cases} 7-2a>0 \rightarrow a < \frac{7}{2} \\ 2a+3>0 \rightarrow a > -\frac{3}{2} \\ a-2>0 \rightarrow a>2 \end{cases} \xrightarrow{\cap} (2, 3/5) \Rightarrow \{3\} \in \mathbb{N}$

$A(9,1)$

$B(1,1)$

جایگذاری : $\begin{matrix}1\\1\end{matrix}$ ✓

$1 = a - 10a + 25a + 3$

$16a + 2 = 0 \rightarrow a = -\frac{1}{8}$

$y = -\frac{1}{8}x^2 + \frac{10}{8}x \underbrace{- \frac{25}{8} + 3}_{-\frac{1}{8}} \xrightarrow{x=0} y = -\frac{1}{8}$

طول : $\left|-\frac{1}{8}\right| = \frac{1}{8}$

سوال ۷)

$\beta \rightarrow a\beta^2 - a\beta - b = 0$

$a(\beta^2-\beta) = b \rightarrow \beta^2 - \beta = \frac{b}{a}$

$S = \frac{-b}{a} = \frac{-(-a)}{a} = 1 \qquad P = \frac{c}{a} = \frac{-b}{a}$

$20\beta^2 + 20\beta^2 + 20\alpha^2 - 20\beta = 17$

$20(\alpha^2+\beta^2) + 20(\beta^2-\beta) = 17$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$20\left(1^2 - 2\left(\frac{-b}{a}\right)\right) + 20\left(\frac{b}{a}\right) = 17$

$20 + 40\frac{b}{a} + \frac{20b}{a} = 17 \rightarrow 60\frac{b}{a} = -3 \rightarrow a = -20b$

$y = -20bx^2 + 20bx - b$

تفاضل ریشه ها $= \frac{\sqrt{\Delta}}{|a|} \rightarrow \frac{\sqrt{400b^2 - 4(-20b)(-b)}}{|-20b|} = \frac{\sqrt{320b^2}}{20|b|} = \frac{4|b|\sqrt{20}}{20|b|} = \frac{2\sqrt{5}}{5}$

سوال ۸)

y ها یکسان → طول رأس سهمی (واقع بر محور تقارن) : $\frac{-5+1}{2} = -2$

$S \left| \begin{matrix} -2 \\ -\frac{1}{2} \end{matrix} \right.$

$\frac{-b}{2a} = -2 \rightarrow b = 4a$

$y = ax^2 + 4ax + \left(\frac{3}{2}\right) \rightarrow \begin{matrix} x=0 \\ y = \frac{3}{2} \end{matrix}$

جایگذاری رأس : $-\frac{1}{2} = 4a - 8a + \frac{3}{2} \rightarrow -4a + 2 = 0 \rightarrow a = \frac{1}{2}$

$y = \frac{1}{2}x^2 + 2x + \frac{3}{2} \xrightarrow{x=1} y = \frac{1}{2}(1)^2 + 2(1) + \frac{3}{2} = 4 \rightarrow \beta = 4$

سوال ۹)

$2/5\alpha^2 + 0/5\alpha^2 + 2/5\beta^2 - 0/5\beta^2 = 2/5\underbrace{(\alpha^2+\beta^2)}_{S^2-2P} + 0/5\underbrace{(\alpha^2-\beta^2)}_{(\alpha+\beta)(\alpha-\beta)} \rightarrow 2/5(36-2a) + 0/5(-6)(-\sqrt{36-4a})$

$\alpha < \beta$

$S = -\frac{b}{a} = -6 \quad , \quad P = \frac{c}{a} = a$

تفاضل : $\frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{36-4a}}{1}$

$= 90 - 5a + 3\sqrt{\underbrace{36-4a}_{4(9-a)}} = 12\sqrt{2} + 85$

$\rightarrow 90 - 5a = 85 \rightarrow \boxed{a=1}$

$\rightarrow 6\sqrt{9-a} = 12\sqrt{2} \rightarrow 9-a = 8$

$a = 1$ ✓

سوال ۱۰)

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{\sqrt{\beta}+\sqrt{\alpha}}{\sqrt{\alpha\beta}} \overset{*}{=} \frac{\sqrt{\frac{m+14}{36} + 2\sqrt{\frac{1}{36}}}}{\sqrt{\frac{1}{36}}} = \frac{\sqrt{\frac{m+14}{36} + 2\left(\frac{1}{6}\right)}}{\frac{1}{6}} = 6\sqrt{\frac{m+24}{36}} = \sqrt{m+24}$

$* \quad \sqrt{(\sqrt{\alpha}+\sqrt{\beta})^2} = \sqrt{\alpha+\beta+2\sqrt{\alpha\beta}}$

$S = -\frac{b}{a} = \frac{m+14}{36}$

$P = \frac{c}{a} = \frac{1}{36}$

$\sqrt{m+24} = 5 \rightarrow m + 24 = 25$

$m = -1$

$mx^2 + 3x + 2 = 0$

$\rightarrow -x^2 + 3x + 2 = 0$

حاصل ضرب $= \frac{c}{a} = \frac{2}{-1} = -2$