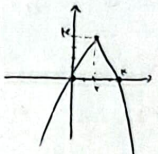
نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۵، کلاس .....

۱

ب) $y=-x^2+4x$

$\text{Max}\begin{cases} x=\frac{-b}{2a}=\frac{-4}{-2}=2 \\ y=-4+8=4 \end{cases}$

$x=0 \longrightarrow y=0$

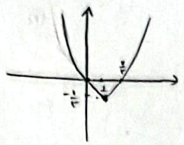


از ناحیه دوم نمی گذرد

الف) $y=3x^2-2x$

$\text{Min}\begin{cases} x=\frac{-b}{2a}=\frac{1}{6}=\frac{1}{3} \\ y=\frac{-\Delta}{4a}=\frac{-4}{12}=\frac{-1}{3} \end{cases}$

$x=0 \longrightarrow y=0$



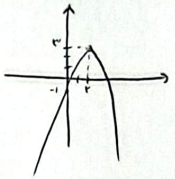
از ناحیه سوم نمی گذرد

۲

ب) $y=-x^2+4x-1$

$\text{Max}\begin{cases} x=\frac{-4}{-2}=2 \\ y=-4+8-1=3 \end{cases}$

$x=0 \longrightarrow y=-1$

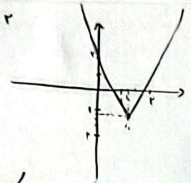


از نواحی اول، سوم و چهارم می گذرد.

الف) $y=2x^2-5x+3$

$\text{Min}\begin{cases} x=\frac{5}{4} \\ y=\frac{-1}{8} \end{cases}$

$x=0 \longrightarrow y=3$



از نواحی اول، دوم و چهارم می گذرد

۳

الف) $S=\alpha+\beta=\frac{-b}{a}=\frac{1}{1}=1 \quad / \quad \alpha-\beta=\frac{\sqrt{\Delta}}{|a|}=\frac{\sqrt{1+12}}{1}=\sqrt{13} \Rightarrow \frac{\alpha+\beta}{\alpha-\beta}=\frac{1}{\sqrt{13}}=\frac{\sqrt{13}}{13}$

ب) $\alpha^2+\beta^2=S^2-2P=1^2-2\left(\frac{-3}{1}\right)=1+6=7$ (۷)

ج) $\alpha^3+\beta^3=S^3-3PS=1^3-3\times(-3)(1)=1+9=10$ (۱۰)

د) $\alpha^3-\beta^3=(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta)=\sqrt{13}\,(7+(-3))=4\sqrt{13}$ (۴√۱۳)

۴

$y=(x-2)(x^2-ax+a)$

حالت ① $x=2 \longrightarrow 4-2a+a=0 \rightarrow 4=a$

حالت ② $x=2$ ریشه ندارد $\longrightarrow \Delta<0 \rightarrow a^2-4a<0 \rightarrow a(a-4)<0$

	0	4	
	+	−	+

$\rightarrow (0,4)$

ریشه ای معادل ندارد

$① \cup ② = (0,4) \cup \{4\} = (0,4]$

۵

$3x^2-12x+a=0 \longrightarrow 3\alpha^2-12\alpha+a=0 \rightarrow 3\alpha^2-12\alpha=-a \rightarrow \alpha^2-4\alpha=\frac{-a}{3}$

$2\alpha^2+\beta^2-4\alpha=7 \longrightarrow \underbrace{\alpha^2+\beta^2}_{S^2-2P}+\alpha^2-4\alpha=7 \rightarrow 16-\frac{2a}{3}-\frac{a}{3}=7 \rightarrow 16-a=7 \rightarrow a=9$

$S=\frac{-b}{a}=\frac{12}{3}=4$

$P=\frac{c}{a}=\frac{a}{3}$

$\Rightarrow 3x^2-12x+9=0 \xrightarrow{a+b+c=0} x=1 \;,\; x=\frac{9}{3}=3$ (۳)

$\frac{a}{\text{ریشه بزرگ}}=\frac{9}{3}=3$ (۳)

۶

$x_{\text{رأس}} = \frac{-b}{2a} = \frac{10}{2} = 5$ ، $y = -\frac{1}{2}(0-5)^2 + 9 = -\frac{25}{2} + \frac{18}{2} = -\frac{7}{2}$ ← عرض راس

$x=0$

$\frac{-1}{2}(0-5)^2+9=y$

ext $\begin{cases} b = 5 \\ b-2 = 4 \end{cases}$

$\Rightarrow K(x-5)^2+9=y \xrightarrow{(1,1)} K(1-5)^2+9=1 \rightarrow 16K=-8 \rightarrow K=-\frac{1}{2}$

مؤلفه طبیعی $\begin{cases} 7-2a>0 \rightarrow 7>2a \rightarrow 3.5>a \\ a-2>0 \rightarrow a>2 \end{cases}$ ← a صحیح

$3.5>a>2 \Rightarrow a=3$

$\Rightarrow (7-2a, a-2) = (1,1)$

۷

$20\beta^2 + 20\beta^4 - 20\beta + 20\alpha^4 = 17 \rightarrow 20(\beta^4+\alpha^4+\beta^2-\beta) = 17$

$S = \frac{-b}{a} = \frac{a}{a} = 1$

$P = \frac{c}{a} = \frac{-b}{a}$

$a\beta^2 - a\beta - b = 0 \rightarrow a(\beta^2-\beta) = b \rightarrow \beta^2-\beta = \frac{b}{a}$ $\quad / \alpha^2+\beta^2 = S^2-2P = 1+\frac{2b}{a}$

$\rightarrow 20\left(1+\frac{2b}{a}+\frac{b}{a}\right) = 17 \rightarrow 20+20\frac{b}{a} = 17 \rightarrow 20\frac{b}{a} = -3 \rightarrow \frac{b}{a} = \frac{-1}{20} \rightarrow b = \frac{-a}{20}$

$1+3\frac{b}{a}$

فاصله ریشه‌ها $= \sqrt{S^2-4P} = \sqrt{1-4\times\frac{-b}{a}} = \sqrt{1+4\frac{b}{a}} = \sqrt{1+4\times\frac{-1}{20}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

۸

ext $\begin{cases} x = \text{طول رأس} = \frac{-10}{2} = -2 = \frac{-b}{2a} \rightarrow b = 4a \\ -\frac{1}{2} \end{cases}$ $\quad c = \frac{3}{2}$

$\Rightarrow y = ax^2+bx+c \rightarrow -\frac{1}{2} = a\times(-2)^2 + b(-2) + \frac{3}{2}$

$y = \frac{1}{2}x^2+2x+\frac{3}{2}$

$-\frac{1}{2} = 4a-2b+\frac{3}{2} \rightarrow 4a-2b = -2$

$2a-b=-1$

$2a-4a=-1$

$-2a=-1$

$a=\frac{1}{2}$ / $b=2$

$(1,\beta)$

$\beta = \frac{1}{2}+2+\frac{3}{2} = \frac{4}{2}+2 = 2+2 = 4$

۹

$x = \frac{-12 \pm \sqrt{144-4a}}{2} = -3 \pm \sqrt{9-a}$ $\xrightarrow{\alpha<\beta}$ $\alpha = -3-\sqrt{9-a}$ ، $\beta = -3+\sqrt{9-a}$

$3\alpha^2 = 3(-3-\sqrt{9-a})^2 = 3(9+9-a+6\sqrt{9-a}) = 54-3a+18\sqrt{9-a}$

$2\beta^2 = 2(-3+\sqrt{9-a})^2 = 2(9+9-a-6\sqrt{9-a}) = 36-2a-12\sqrt{9-a}$

$3\alpha^2+2\beta^2 = 90-5a+6\sqrt{9-a} \rightarrow 90-5a+6\sqrt{9-a} = 12\sqrt{2}+57$

$\rightarrow 33-5a+6\sqrt{9-a} = 12\sqrt{2} \rightarrow (a=1)$

$\sqrt{8}+3\sqrt{2} = 5\sqrt{2} = 12\sqrt{2}$ ✓

۱۰

$\left(\sqrt{\frac{1}{\alpha}}+\sqrt{\frac{1}{\beta}}=5\right)^2 \rightarrow 25 = \frac{1}{\alpha}+\frac{1}{\beta}+2\sqrt{\frac{1}{\alpha\beta}} = \frac{\alpha+\beta}{\alpha\beta}+2\sqrt{\frac{1}{\alpha\beta}}$

$S = \alpha+\beta = \frac{-b}{a} = \frac{m+14}{3}$

$P = \frac{c}{a} = \alpha\beta = \frac{1}{3}$

$= \frac{\frac{m+14}{3}}{\frac{1}{3}} + 2\sqrt{\frac{1}{\frac{1}{3}}} = m+14+12 = 25$

$m+26 = 25$

$m=-1$

$\Rightarrow -m^2+3m+2=0 \rightarrow P = \frac{c}{a} = \frac{2}{-1} = -2$