محمدمهدی حاجی اکبری

سوال ۱)

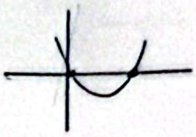
الف) $3x^2 - 2x$

$\Delta = b^2 - 4ac = 4 - 4(3)(0) = 4$

$S = \frac{-b}{a} = \frac{2}{3}$

$P = \frac{c}{a} = 0 \rightarrow$

$\frac{-b \pm \sqrt{\Delta}}{2a} = \frac{2 \pm \sqrt{4}}{6} = 0 , \frac{2}{3}$



$\Rightarrow$ از ناحیه ۳ نمی‌گذرد

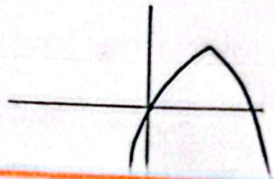
ب) $-x^2 + 4x$

$\Delta = b^2 - 4ac = 16 - 0 = 16$

$S = \frac{-b}{a} = \frac{-4}{-1} = 4$

$P = \frac{c}{a} = 0$

$\Rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-4 \pm \sqrt{16}}{-2} \rightarrow 0 , 4$



$\Rightarrow$ از ناحیه ۲ نمی‌گذرد

سوال ۲)

الف) $2x^2 - 5x + 2$

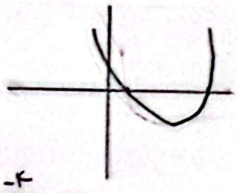
$\Delta = b^2 - 4ac = 25 - 4(2)(2) = 9$

$S = \frac{-b}{a} = \frac{5}{2}$

$P = \frac{c}{a} = 1$

$x = \frac{-b \pm \sqrt{\Delta}}{2a}$

$x = \frac{5 \pm \sqrt{9}}{4} \rightarrow 2 , \frac{1}{2}$



از ناحیه ۲، ۳ و ۴ نمی‌گذرد

ب) $-x^2 + 4x - 1$

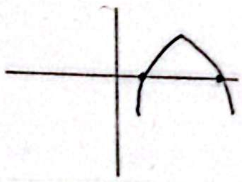
$\Delta = b^2 - 4ac = 16 - 4(-1)(-1) = 12$

$S = \frac{-b}{a} = \frac{-4}{-1} = 4$

$P = \frac{c}{a} = 1$

$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-4 \pm \sqrt{12}}{-2}$

$x \rightarrow 2 - \sqrt{3} , \; 2 + \sqrt{3}$



از ناحیه ۲ و ۳ نمی‌گذرد

سوال ۳) $x^2 - x - 3 = 0$

الف) $\frac{\alpha + \beta}{\alpha - \beta} = \frac{\frac{-b}{a}}{\frac{\sqrt{\Delta}}{|a|}} = \frac{\frac{+1}{1}}{\sqrt{1 - 4(1)(-3)}} = \frac{1}{\frac{\sqrt{13}}{1}} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

ب) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (\frac{-b}{a})^2 - 2(\frac{c}{a}) = 1 - 2(-3) = 7$

ج) $(\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) = (\frac{-b}{a})(S^2 - 2P - \frac{c}{a}) = \frac{1}{1}(7 + 3) = 10$

د) $(\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta) = (\sqrt{13})(7 + (-3)) = \sqrt{13}(4) = 4\sqrt{13}$

سوال ۴)

$(x - 2)(x^2 - ax + a) = 0$

$\Delta < 0 \Rightarrow b^2 - 4ac < 0 \Rightarrow a^2 - 4(1)(a) < 0 \Rightarrow a^2 - 4a < 0 \quad a(a - 4) < 0$

$+ \; 0 \; - \; 4 \; + \Rightarrow a \in (0, 4]$

۲) $(x - 2)^2 = x^2 + 4 - 4x \Rightarrow a = 4$

سوال ۵)

$x^2 - 4x = \frac{a}{3}$ $\qquad S^2 - 2P = (\frac{-b}{a})^2 - 2(\frac{c}{a}) = (\frac{12}{3})^2 - 2(\frac{-a}{3})$

$= 16 + \frac{2a}{3}$

$\alpha^2 + \underbrace{\alpha^2 + \beta^2}_{S^2-2P} - 4\alpha = 7$

$\frac{a}{3} + 16 + \frac{2a}{3} = 7 \Rightarrow 16 + a = 7 \Rightarrow a = -9$

$3x^2 - 12x + 9 = 0 \Rightarrow x^2 - 4x + 3 = 0$ $\quad 1 \to \pm\frac{c}{a} = +3 \to$ ریشه‌ها ، $3 \to +1$

$\frac{a}{3} = \frac{-9}{3} = -3$ جواب

سوال ۶)

رأس $= \frac{7 - 2a + 2a + 3}{2} = 5$ $\qquad \begin{vmatrix} b \\ b-2 \end{vmatrix} \Rightarrow \begin{vmatrix} 5 = h \\ 3 = k \end{vmatrix}$ $\qquad y = z(x-h)^2 + k$

$y = z(x-5)^2 + 3$

$y = -\frac{1}{8}(x-5)^2 + 3$

جایگذاری $\quad a - 2 = z(2a + 3 - 5)^2 + 3$

$* \to 1 = z(4 + (-2) + 3)^2 \Rightarrow 16z = -2 \Rightarrow z = -\frac{1}{8}$

$y = -\frac{1}{8}(x^2 + 25 - 10x) + 3$

$*$ if $a = 3 \to A|1$
$\qquad\qquad \to B|9$

$y = -\frac{1}{8}x^2 + \frac{5}{4}x - \frac{25}{8} + 3$

$c = -\frac{25}{8} + \frac{24}{8} = -\frac{1}{8}$

$\sqrt{(0-0)^2 + (0 - (-\frac{1}{8}))^2} = \frac{1}{8}$ = فاصله

سوال ۷)

$\alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{a}{a} = 1 \Rightarrow \beta = 1 - \alpha$

$40\beta^2 + 20\alpha^2 - 20\beta = 17 \overset{\div 20}{\Rightarrow} 2\beta^2 + \alpha^2 - \beta = \frac{17}{20}$

$2(1-\alpha)^2 + \alpha^2 - (1-\alpha) = \frac{17}{20} \Rightarrow 2 + 2\alpha^2 - 4\alpha + \alpha^2 - 1 + \alpha = \frac{17}{20}$

$60\alpha^2 - 60\alpha + 20 = 17 \Rightarrow 60\alpha^2 - 60\alpha + 3 = 0 \quad \Leftarrow 3\alpha^2 - 3\alpha + 1 = \frac{17}{20}$

$20\alpha^2 - 20\alpha + 1 = 0 \Rightarrow \alpha = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{20 \pm \sqrt{320}}{40} \Rightarrow \alpha = \frac{5 \pm 2\sqrt{5}}{10}$

$\beta = 1 - \alpha \Rightarrow \beta = 1 - (\frac{5 \pm 2\sqrt{5}}{10}) \Rightarrow \beta = \frac{-5 \pm 2\sqrt{5}}{10}$

$\frac{\sqrt{\Delta}}{|a|} \qquad |\alpha - \beta| = \alpha - (1 - \alpha) = 2\alpha - 1 = \left|2(\frac{5 \pm 2\sqrt{5}}{10})\right| = \frac{2\sqrt{5}}{5}$

سوال ۸)

طول رأس $= \frac{1-5}{2} = -2$ $\qquad$ رأس $\begin{vmatrix} -2 = h \\ -\frac{1}{2} = k \end{vmatrix} \longrightarrow y = a(x+2)^2 - \frac{1}{2}$

$y = \frac{2}{9}(x-3)^2 - \frac{1}{2}$ $\qquad x=0: \frac{3}{2} = 4a - \frac{1}{2} \Rightarrow 4a = 2 \Rightarrow a = \frac{1}{2}$

$y = \frac{1}{2}(x+2)^2 - \frac{1}{2} \Rightarrow y = \frac{1}{2}x^2 + 2 + 2x - \frac{1}{2} \Rightarrow y = \frac{1}{2}x^2 + 2x + \frac{3}{2}$

جایگذاری $x = 1$: $\frac{1}{2} + 2 + \frac{3}{2} = 4 = \beta$

سؤال ۹)

$2\alpha^2+\alpha^2+2\beta^2 = 2(\alpha^2+\beta^2)+\alpha^2$

$\alpha+\beta = \frac{-b}{a} = -4 \Rightarrow \beta = -4-\alpha$

$3\alpha^2+3\beta^2 = 3\alpha^2+2(-4-\alpha)^2 \Rightarrow 3\alpha^2+3\alpha^2-24\alpha+32 = 12\sqrt{2}+18$

$3\alpha^2+16+\alpha^2+8\alpha$ $\qquad 6\alpha^2+24\alpha+32 = 12\sqrt{2}+18$

$6\alpha^2+24\alpha-12 = 12\sqrt{2}$ $\xrightarrow{\text{طرفین بر } 6}$ $\alpha = -2-2\sqrt{2}$, $\beta = -2+2\sqrt{2}$ $\Big\}$ $\alpha\beta = a \to 9-8 = a \Rightarrow a = 1$

سؤال ۱۰)

$\alpha+\beta = \frac{m+14}{3q}$ $\qquad \alpha\beta = \frac{1}{3q}$

$\sqrt{\frac{1}{\alpha}}+\sqrt{\frac{1}{\beta}} = 5 \xrightarrow{\text{به توان } 2} \frac{1}{\alpha}+\frac{1}{\beta}+2\sqrt{\frac{1}{\alpha\beta}} = 25$

$\frac{\alpha+\beta}{\alpha\beta}+2\sqrt{\frac{1}{\alpha\beta}} = 25$

$\frac{\frac{m+14}{3q}}{\frac{1}{3q}} = 15$ $\qquad \frac{\frac{m+14}{3q}}{\frac{1}{3q}} = 15$

$m+14 = 15 \Rightarrow m = -1$

$3q^2-15q+1 = 0$ $\qquad -x^2+8x+2$ $\xrightarrow{\text{حاصل ضرب}}$ $\frac{c}{a} = -2$