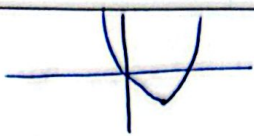

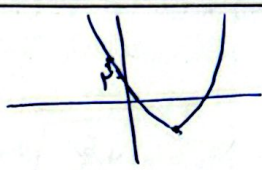
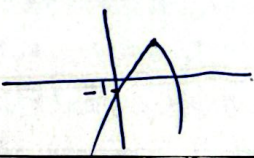


الف) $y = 3x^2 - 2x \rightarrow \min \left| \begin{array}{l} -\frac{b}{2a} = \frac{1}{3} \\ -\frac{1}{3} \end{array} \right.$  از ناصبه ۳ می گذرد.

ب) $y = -x^2 + 2x \rightarrow \max \left| \begin{array}{l} -\frac{b}{2a} = 1 \\ 2 \end{array} \right.$  از ناصبه ۱ می گذرد.

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الف) $y = 2x^2 - 2x + 2 \rightarrow \min \left| \begin{array}{l} -\frac{b}{2a} = \frac{1}{2} \\ -\frac{1}{2} \end{array} \right.$  از ناصبه ۱ و ۳ می گذرد.

ب) $y = -x^2 + 2x - 1 \rightarrow \max \left| \begin{array}{l} -\frac{b}{2a} = 1 \\ -1 \end{array} \right.$  از ناصبه ۱ و ۳ می گذرد.

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$x^2 - m - 3 = 0 \rightarrow S = 1 \quad / \quad P = -3 \quad / \quad |\alpha - \beta| = \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{\sqrt{13}}{1} = \sqrt{13}$

الف) $\frac{\alpha + \beta}{\alpha - \beta} = \frac{S}{|\alpha - \beta|} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

ب) $\alpha^2 + \beta^2 = S^2 - 2P = 1 - 2(-3) = 7$

ج) $\alpha^2 + \beta^2 = S^3 - 3SP = 1 - 3(-3) = 10$

د) $\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta) = 13\sqrt{13} + 3(-3)(\sqrt{13}) = 4\sqrt{13}$

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$y = (m-2)(m^2 - am + a) \rightarrow m = 2 \rightarrow a^2 - 2a < 0 \rightarrow a(a-2) < 0$

① $0 < a < 2$

$\Delta \geq 0 \rightarrow (m-2)^2 = a^2 - 2m + 2 \xrightarrow{m=2} a = 2$

① U ② : $0 < a \leq 2$

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$3x^2 - 12x - a = 0 \rightarrow S = \alpha + \beta = 4 \rightarrow \beta = 4 - \alpha$

$2\alpha^2 + \beta^2 - 2\alpha < 7 \rightarrow 2\alpha^2 + (4 - \alpha)^2 - 2\alpha - 7 < 7 \rightarrow \alpha^2 - 2\alpha + 4 = 0 \rightarrow (\alpha - 1)(\alpha - 3) = 0$

$\rightarrow \alpha = 1, 3$

$\hookrightarrow 3x^2 - 12x - a = 0 \rightarrow a = -9 \rightarrow \frac{9}{3}, \frac{-9}{3} = -3$

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$$\begin{aligned}
 a-r \in \mathbb{N} &\rightarrow a-r > 0 \rightarrow a > r \\
 v-r \in \mathbb{N} &\rightarrow v-r > 0 \rightarrow a < v \\
 &\rightarrow S(a, r) \\
 \rightarrow 1=Na+rb+c &\left\{ \begin{aligned} 1=Na+rb > 0 &\rightarrow 1=a+rb \rightarrow b = -1+a \\ 1=Na+rb+c &\rightarrow 1=Na+rb+c \end{aligned} \right. \\
 1=Na+rb+c &\rightarrow 1=Na+rb+c \\
 r=ra+a \cdot a(-1+a)+c+r=ra+a \cdot a+c &\rightarrow ra+a^2=c-r \\
 \rightarrow a = \frac{1}{r} &\rightarrow \frac{1}{r} = c-1 \rightarrow a = \frac{1}{r} \rightarrow \text{Mod } \frac{1}{r}
 \end{aligned}$$

$$\begin{aligned}
 a\alpha^r - a\alpha - b = 0 &\rightarrow \alpha + \beta = 1 \mid \alpha\beta = \frac{-b}{a} \\
 \Sigma \beta^r = r \cdot \alpha^r - r \cdot \beta = 1 &\rightarrow \Sigma (1-\alpha)^r + r \cdot \alpha^r - r \cdot (1-\alpha) = 1 \\
 -\alpha^r - \alpha + \frac{1}{r} = 0 &\rightarrow \Delta = \frac{r}{4} \\
 \rightarrow |\alpha_1 - \alpha_2| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{\frac{r}{4}}}{1} = \frac{r}{\sqrt{a}} = \frac{r\sqrt{a}}{a}
 \end{aligned}$$

$$\begin{aligned}
 \text{Or } \frac{1}{r} = \frac{-a+1}{r} = x &\rightarrow \text{ent } \frac{1}{r} \rightarrow y = -\frac{1}{r}, c = \frac{r}{r} \\
 y = a\alpha^r + b\alpha = \frac{r}{r} &\rightarrow \Sigma a - rb + \frac{r}{r} = \frac{1}{r} \rightarrow b - rb = -\frac{1}{r} - \frac{r}{r} \rightarrow b = r, a = \frac{1}{r} \\
 raa - ab + \frac{r}{r} = a + b + \frac{r}{r} &\rightarrow \epsilon a = b \\
 \rightarrow \frac{1}{r} \alpha^r + r\alpha + \frac{r}{r} = y &\rightarrow \frac{1}{r} \alpha^r + r\alpha + \frac{r}{r} = \frac{1}{r} \rightarrow \alpha^r + r\alpha = 0 \rightarrow \boxed{\beta = \frac{1}{r}}
 \end{aligned}$$

$$\begin{aligned}
 \alpha^r + r\alpha = a = 0 &\rightarrow \alpha = -r + \sqrt{r^2 - a} \rightarrow \alpha^r = 1 - a - 4\sqrt{r^2 - a} \\
 \beta = -r - \sqrt{r^2 - a} &\rightarrow \beta^r = 1 - a + 4\sqrt{r^2 - a} \\
 \rightarrow r\alpha^r + r\beta^r = 9 \cdot -2a - 4\sqrt{r^2 - a} = 12\sqrt{r^2 - a} + 18 &\rightarrow 2a + 4\sqrt{r^2 - a} = 9 + 9\sqrt{r^2 - a} \rightarrow \boxed{a = 1}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}} = 2 &\rightarrow \frac{\sqrt{a} + \sqrt{b}}{\sqrt{ab}} = 2 \rightarrow \sqrt{a} + \sqrt{b} = 2\sqrt{ab} \\
 S + r\sqrt{p} = rA + P = S + r\sqrt{\frac{1}{r^2}} = \frac{rA}{r^2} &\rightarrow S = \frac{14}{r^2} \rightarrow \frac{r+12}{r^2} = \frac{14}{r^2} \\
 \rightarrow m = -1 &\rightarrow m\alpha^r + r\alpha + r = -m\alpha^r + r\alpha + r \rightarrow p = \frac{c}{a} = \frac{r}{1} = \boxed{-r}
 \end{aligned}$$