۱- سوال اساسی

الف) $\begin{cases} 2x - y = 9 \\ x + 2y = -4 \end{cases}$ $\begin{cases} 9x - 2y = 18 \\ x + 2y = -4 \end{cases}$

$7x = 14 \Rightarrow x = 2 \quad y = -3$

$2y = -4 - 2$

$\frac{x}{y} = \frac{-2}{3}$

ب) $\frac{1}{x} - \frac{1}{y} = -1 \rightarrow \frac{y - x}{xy} = -1$

$\frac{a}{x} - \frac{7}{y} = \frac{ay - 7x}{xy} = -3$

$y - x = -xy$

$ay - 7x = -3xy$

$3y - 3x = ay - 7x$

$4x = 2y \quad y = 2x$

$\frac{x}{y} = \frac{x}{2x} = \frac{1}{2}$

۲-

$2a + 2b = 3a + 3$

$2b = a + 3$

$b = \frac{a+3}{1} = \frac{-3+3}{2} = 0$

$a + 1 = -2$

$a = -3$

۳-

$m^2 - 3m = -2$

$m^2 - 3m + 2 = 0$

$(m-1)(m-2) = 0$

$m = 1$

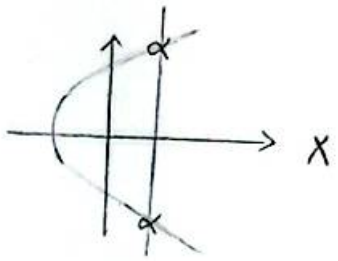
$f = \{(-1,-2), (3,a), (-1,-2), (2,4), (2,4), (3,a)\}$

$m = 2$

$f = \{(-1,-2), (3,a), (-1,-2), (3,4), (2,4), (4,9)\}$ ✗

۴- برای هیچ مقدار m

الف)

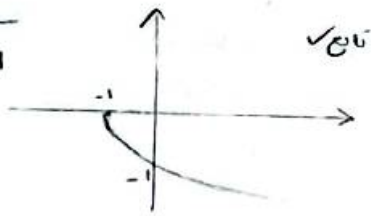


ب) ✓

ج)

۵-

الف) $y = -\sqrt{x+1}$



تابع ✓

ب) $x = \frac{2}{\sqrt{1-y^2}}$

$\frac{2_1}{\sqrt{1-y_1^2}} = \frac{2_2}{\sqrt{1-y_2^2}}$

$y_1^2 - y_1^2 y_2^2 = y_2^2 - y_1^2 y_2^2 \quad y_1 = y_2$ هم علامت

$\frac{y_1^2}{1-y_1^2} = \frac{y_2^2}{1-y_2^2}$

$y_1 = y_2$ ∴ تابع هست

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الف) $|x|=1 \xrightarrow{x<0} x=\pm 1$ تابع نیست

ب) $y^3+3y^2+3y+x^3+x=0 \quad y_1^3+3y_1^2+3y_1=y_2^3+3y_2^2+3y_2 \rightarrow$

$y_1^3-y_2^3=3y_2^2-3y_1^2+3y_2-3y_1 \rightarrow (y_1-y_2)(y_1^2+y_1y_2+y_2^2)=3(y_2-y_1)(y_2+y_1+1)$

$(y_1-y_2)(y_1^2+y_1y_2+y_2^2+3y_2+3y_1+3)=0 \rightarrow (y_1-y_2)(y_1^2+y_1y_2+y_2^2+3y_2+3y_1+3)=0$

$\begin{cases} ① \ y_1-y_2=0 \rightarrow y_1=y_2 \\ ② \ y_1^2+y_1(y_1+3)+y_1^2+3y_1+3=0 \rightarrow \Delta=(y_1+3)^2-4(y_1^2+3y_1+3)=y_1^2+9+6y_1-4y_1^2-12y_1 \end{cases}$

$-12=-3y_1^2-6y_1-3 \longrightarrow -3(y_1^2+2y_1+1)=-3(y_1+1)^2 \rightarrow \Delta<0$ $\begin{cases} ① \ \Delta<0 \text{ تابع} \checkmark \\ \Delta=0 \end{cases}$ بقیه حتمی عبارت دوم تابع ✓

$y_1^2+2y_1+1=0 \quad y_1=-1$

$y_1=-1=y_2$ تابع ✓

۷

$f(x)=\dfrac{x^2+4x+5}{x^2+4x+7}=\dfrac{(x+2)^2+1}{(x+2)^2+3} \xrightarrow{x=\sqrt{3}-2}$

$\dfrac{(\sqrt{3}-2+2)^2+1}{(\sqrt{3}-2+2)^2+3}=\dfrac{3+1}{3+3}=\dfrac{4}{6}=\boxed{\dfrac{2}{3}}$

۹

$f(x)=x^3+ax+b \quad y=3x-a$ $\Big\}$ $\xrightarrow{\begin{matrix}1 \\ -4\end{matrix}}$ $-4=-3-a \rightarrow a=1$

$-4=-1-1+b \rightarrow b=-2$

$x^3-2x-1=0 \rightarrow x(x-1)(x+1)-(x+1)=0$

$(x+1)(x^2-x-1)=0 \Rightarrow \begin{cases} ① \ x+1=0 \rightarrow x=-1 \\ ② \ x^2-x-1=0 \end{cases}$ این نقطه داده شده و ما نقطه دیگر را می خواهیم

$\delta=-\dfrac{b}{a}=\boxed{1}$

۱۰

$f=\{(2,a+b),(1,2a),(-1,a-2b+1)\}$ ①,② $\Rightarrow a+b=2a \quad a=b$

①,③ $\rightarrow a-2b+1=2a \rightarrow 1=3a \quad \boxed{a=\dfrac{1}{3}}$

۳

$f(x)=\dfrac{4x^2-ax+c+1}{bx+3}=x \quad 4x^2-ax+c+1=bx^2+3x \rightarrow x^2(4-b)+x(-a-3)+c+1=0$

$4-b=0 \quad \boxed{b=4}$ $\quad c+1=0 \quad \boxed{c=-1}$ $\quad a+b+c=\boxed{0}$

$-a-3=0 \rightarrow \boxed{a=-3}$