

۱۸, ۱۵۵

$$f(x) = \begin{cases} x^2 + 2x & x \geq a \\ ax - 2 & x < a \end{cases} \quad x=a \Rightarrow a^2 + 2a = a^2 - 2$$

$$\boxed{2a = -2}$$

۵

$$f(x) = \frac{x^2 + a}{2x - b} \quad g(x) = 2x + b$$

$$(2, 2) \Rightarrow \frac{2+a}{2-b} = 2 \quad \begin{cases} 2+a = 2(2-b) \\ a + 2b = 1 \end{cases} \quad \begin{matrix} 2+b = 2 \\ b = -1 \end{matrix} \quad a = 11$$

$$f(x) = \frac{x^2 + 11}{2x + 1} \Rightarrow f(1) = \frac{12}{3} = \boxed{4}$$

۵

$$f(x) = \frac{2x+1}{2x^2+ax+b} \Rightarrow \begin{cases} x = -1 \\ x = 2 \end{cases} \quad \begin{cases} 2x^2+ax+b = 0 \\ 2x^2+ax+b = 0 \end{cases}$$

$$\begin{cases} 2-a+b = 0 \\ 2+2a+b = 0 \end{cases} \Rightarrow \begin{matrix} 2a = -4 \\ a = -2 \\ b = -1 \end{matrix}$$

$$f(x) = \frac{2x+1}{2x^2-4x-1} \quad f(1) = \frac{3}{-1} = \boxed{-\frac{3}{1}}$$

۵

$$f(x) = \frac{x^2 - \sqrt{2}}{-2x^2 + ax + b}$$

$$-2x^2 + ax + b = -2(x+1)^2 = -2x^2 - 4x - 2$$

$$\begin{matrix} a = -4 \\ b = -2 \end{matrix}$$

$$a+b = -4-2 = \boxed{-6}$$

۵

$$f(x) = \frac{2x}{(x-1)(x^2+mx+1)}$$

$$x^2+mx+1 \Rightarrow \Delta < 0$$

$$m^2 - 4 < 0$$

$$m^2 < 4 \Rightarrow -2 < m < 2 \quad I$$

حالت ۲ ←  $x=1$  در صورتی باشد

$$(x-1)^2 = x^2 + mx + 1 \Rightarrow x^2 - 2x + 1 = x^2 + mx + 1$$

$$m = -2 \quad II$$

جواب:  $\boxed{(-2, 2)}$

۱۸, ۱۵۵

$$f(x) = \sqrt{x - \frac{1}{x^p}} \quad x - \frac{1}{x^p} \geq 0$$

$$x \neq 0$$

$$\frac{x^p - 1}{x^p} \geq 0$$

$$\frac{(x-1)(x+1)}{x^p} \geq 0$$

$$\frac{-1}{+} \quad \frac{+}{-} \quad \frac{+}{+}$$

$$D_f = (-\infty, \frac{1}{p}] \cup [\frac{1}{p}, \infty)$$

6

5

$$f(x) = \sqrt{mx^2 + px + 1}$$

$$mx^2 + px + 1 \geq 0$$

$$\Delta \leq 0 \quad pm^2 - p^2 \leq 0 \quad m > 0$$

$$m(m-1) \leq 0$$

$$\frac{0}{+} \quad \frac{1}{-} \quad \frac{+}{+}$$

$$Zf = [0, 1]$$

$$m=0 \Rightarrow f(x) = 1$$

7

5

$$f(x) = \begin{cases} \frac{x^p - 1}{x^p - 1} & x \neq a \\ x^p + k & x = \frac{1}{p} \end{cases} \quad g(x) = x^p + 1$$

$$\frac{x^p - 1}{x^p - 1} = \frac{(x-1)(x+1)}{x^p - 1} = g(x) \quad x \neq \frac{1}{p} \quad a = \frac{1}{p}$$

$$x = \frac{1}{p} \Rightarrow g(x) = x^p + 1 = x^p + k = x^p + k \quad k = 0 \quad a + k = \frac{1}{p}$$

$$a + k = \frac{1}{p}$$

8

5

$$f(x) = \begin{cases} \frac{px^2 - r}{px + r} & x \neq -\frac{r}{p} \\ px + r & x = -\frac{r}{p} \end{cases} \quad g(x) = px + b$$

$$\frac{px^2 - r}{px + r} = \frac{(px - r)(px + r)}{px + r} = px - r \quad x \neq -\frac{r}{p} \quad b = -r$$

$$g(-\frac{r}{p}) = -r - r = -2r = -pa + r \quad pa = 4 \quad a = p$$

$$a - b = r + r = 2r$$

9

5

$$f(x) = \begin{cases} \frac{x^p - r}{x - r} & x \neq r \\ pa^p + pa & x = r \end{cases} \quad g(x) = x + r$$

$$g(r) = r + r = 2r$$

$$pa^p + pa = r$$

$$a^p + a - r = 0$$

$$\begin{matrix} a = 1 \\ a = -r \end{matrix}$$

$$\frac{x^p - r}{x - r} = \frac{(x - r)(x + r)}{x - r} = x + r \quad x \neq r$$

10

5