بیسنا قراقانی — تکلیف ۲۸ — (هم زنتر)

۲۰

①

$$f(x) = \begin{cases} x^2+2x & x \geq a \\ ax-4 & x < a \end{cases} \quad \xrightarrow{x=a} \quad x^2+2x = ax-4$$

$$a^2+2a = a^2-4$$

$$\boxed{a=-2}$$

②

$$f(x) = \frac{x^2+a}{2x-b} \qquad g(x) = 2x+b \qquad x=1 \Rightarrow f(1) = \frac{1+11}{2+1} = \frac{12}{3} = \boxed{4}$$

$$(2,3) \xrightarrow[x=2]{} f(x) = \frac{4+a}{4-b} = 3 \qquad g(x) = 4+b = 3 \quad x=2 \quad \boxed{b=-1}$$

$$4+a = 12-3b$$

$$a+3b = 8$$

$$\boxed{a=11}$$

③

$$f(x) = \frac{4x+1}{2x^2+ax+b} \qquad f(1) = \frac{4+1}{2-6-8} = \frac{5}{-12}$$

$$R-\{-1,4\} \longrightarrow$$

$$x=-1 \quad 2-a+b=0 \rightarrow b-a=-2$$

$$x=4 \quad 32+4a+b=0 \rightarrow 4a+b=-32$$

$$5a=-30$$

$$a=-6$$

$$b=-8$$

④

$$f(x) = \frac{x^3-\sqrt{3}}{-4x^2+ax+b} \qquad \boxed{a+b=-12}$$

$$R-\{-1\} \longrightarrow x=-1 \quad -4-a+b$$

$$(x+1)^2 = x^2+2x+1$$

$$\times -4$$

$$-4x^2-8x-4$$

$$-4x^2+ax+b \rightarrow a=-8, \quad b=-4$$

(8)

$$f(x)=\begin{cases}\dfrac{4x^2-1}{2x-1} & x\neq a\\ 4x+k & x=\dfrac{1}{2}\end{cases}$$

$$\frac{(2x)^2-1^2}{2x-1}=\frac{(2x-1)(2x+1)}{2x-1}=2x+1$$

$2x-1\neq 0 \xrightarrow{a} 2a-1\neq 0$

$2a\neq 1$

$a\neq \frac{1}{2}$

$x=a=\frac{1}{2}$

$g(x)=2x+1$

$\xrightarrow{\frac{1}{2}} g(\frac{1}{2})=f(\frac{1}{2})$

$1+1=2+k$

$k=0$

(9)

$$f(x)=\begin{cases}\dfrac{9x^2-4}{3x+2} & x\neq -\dfrac{2}{3}\\ 3ax+2 & x=-\dfrac{2}{3}\end{cases}$$

$$\rightarrow \frac{(3x)^2-2^2}{3x+2}=\frac{(3x-2)(3x+2)}{3x+2}=3x-2$$

$3x+2\neq 0$

خودشون برابر باشن

$g(x)=3x+b$ → [illegible] → $f(-\frac{2}{3})=g(-\frac{2}{3})$, $x=-\frac{2}{3}$

$-2a+2=-2+b$ ($b=-2$)

$-2a+2=-4$

$-2a=-6$

$a=3$

$3x-2$

$g(x)=3x+b$

$b=-2$

$a-b=3-(-2)=5$

(10)

$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & x\neq 2\\ 2a^2+ax & x=2\end{cases}$$

$$\rightarrow \frac{(x+2)(x-2)}{(x-2)}=x+2$$

$g(x)=x+2$

$\xrightarrow{x=2} g(2)=f(2)$

$2+2=2a^2+2a$

$4=2a^2+2a$

$2a^2+2a-4=0$

$a^2+a-2=0$

$a+b+c=0$

$a=1$, $a=\frac{c}{a}=\frac{-2}{1}=-2$

⑤

$$f(x) = \frac{2x}{(x-1)(x^2+mx+1)}$$

$R - \{1\}$ → که این تابع دیگر ریشه را نداشته باشد

$\Delta = b^2 - 4ac \rightarrow \Delta = m^2 - 4 < 0$

$m^2 < 4$

$-2 < m < 2$

⑥

$$f(x) = \sqrt{4 - \frac{1}{x^2}}$$

$x \neq 0$

$4 - \frac{1}{x^2} \geq 0 \rightarrow 4 \geq \frac{1}{x^2}$

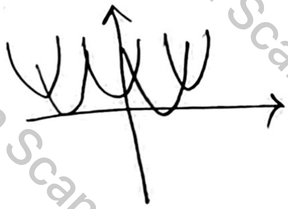
$-2 \leq \frac{1}{x} \leq 2 \xrightarrow{\times x} -2x \leq 1 \leq 2x$

$\begin{cases} -2x \leq 1 \rightarrow x \geq -\frac{1}{2} \\ 1 \leq 2x \rightarrow x \geq \frac{1}{2} \end{cases}$

$D = (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, +\infty)$

⑦

$f(x) = \sqrt{mx^2 + \underset{S}{\underline{2mx}} + \underset{P}{\underline{1}}}$



$\begin{cases} \Delta \leq 0 \rightarrow 4m^2 - 4m \leq 0 \\ m > 0 \\ S = \frac{-b}{a} = \frac{-2m}{m} = -2 \\ P = \frac{c}{a} = \frac{1}{m} \end{cases}$

$4m(m-1) \leq 0$, 0, 1

0		1	
+	−	+	

$m \in [0, 1]$