۱۷

① ص

$a^2+2a=a^2-4 \to a=-2$

② ص

$\frac{x^2+a}{2x-b} \to x=2$ $\quad \frac{4+a}{4-b}=3$ $\quad 2x+b \Rightarrow x=2$ $\quad 4+b=3 \Rightarrow b=-1$

$\hookrightarrow \frac{4+a}{4+1}=3 \Rightarrow a=11$ $\quad f(1)=\frac{1+11}{2-(-1)}=\frac{12}{3}=4$

③ ص

$2-a+b=0 \to b-a=-2 \Rightarrow -30=5a \to a=-6,\ b=-8$

$32+4a+b=0 \to b+4a=-32$ $\quad f(1)=\frac{4+1}{2-6-8}=\frac{-5}{12}$

④ ص

$-4-a+b=0$ $\quad \Delta=0$ $\quad a^2+16b=0$ $\quad b+a=-12$

$b-a=4 \to b=4+a$ $\quad a^2+16a+64=0 \Rightarrow a=-8,\ b=-4$

⑤ ۰/۵

$x^2+mx+1 \longrightarrow$ ریشه ندارد $\quad m^2-4<0 \Longrightarrow$ ~~$-2\le m<2$~~ $-2<m<2$ I

⑥ ۰/۵

$4 \ge \frac{1}{x^2} \longrightarrow \frac{1}{x^2}\le 4 \longrightarrow -2\le$ ~~$\frac{1}{x}\le 2 \Rightarrow$~~ $-\frac{1}{2}\le x\le \frac{1}{2}$

⑦ ص

$f(x)=\sqrt{mx^2+2mx+1}$ $\quad D_f=\mathbb{R}$ $\quad mx^2+2mx+1 \to \Delta=0$

$b^2-4ac=0 \to 4m^2-4m=0 \to m=1$ $\qquad a>0 \to m>0$

$mx^2+2mx+1 \to \Delta<0 \to b^2-4ac<0 \to 4m^2-4m<0 \to m<1$

$m=(0,1]$

⑧ ص

$f(x)=\begin{cases}\frac{4x^2-1}{2x-1} & x\neq a\\ 4x+k & x=\frac{1}{2}\end{cases}$ $\qquad g(x)=2x+1$ $\qquad a+k=\frac{1}{2}+0=\frac{1}{2}$

$2x-1=0 \Rightarrow x=\frac{1}{2} \Rightarrow a=\frac{1}{2}$ $\qquad g(\frac{1}{2})=2$ $\qquad 2+k=2 \Rightarrow k=0$

⑨ ص

$f(x)=\begin{cases}\frac{9x^2-4}{3x+2} & x\neq -\frac{2}{3}\\ 3ax+2 & x=-\frac{2}{3}\end{cases}$ $\qquad g(x)=3x+b$

$g(1),\ f(1):\ 3+b\ ,\ 1 \Rightarrow 3+b=1 \Rightarrow b=-2$ $\qquad f(-\frac{2}{3})=-2a+2$, $\ g(-\frac{2}{3})=-4 \Rightarrow a=3$

$a-b=3-(-2)=5$

(10) ②

$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & x\neq 2\\ 2a^2+ax & x=2\end{cases}\qquad g(x)=x+2$$

$$f(2)=2a^2+2a$$

$$g(2)=4$$

$$\longrightarrow a^2+a=2 \Rightarrow a=\boxed{-2 , 1}$$

حل سوال ۵

$x=1$ جواب معادله تکراری باشد (ریشه مضاعف عبارت باشد)

$$(x-1)^2 = x^2+mx+1$$

$$x^2-2x+1 = x^2+mx+1 \rightarrow m=-2 \quad II$$

$$I \cup II \rightarrow [-2,2)$$

سوال ۷

$$4-\frac{1}{x^2} \geq 0 \rightarrow \frac{4x^2-1}{x^2} \geq 0$$

$$4x^2-1 \geq 0 \rightarrow x^2 \geq \frac{1}{4} \rightarrow x \geq \frac{1}{2}, \quad x \leq -\frac{1}{2}$$

$$D_f = (-\infty, -\tfrac{1}{2}] \cup [\tfrac{1}{2}, +\infty)$$