

۱۸، ۱۲۵

الف) $y^3 = 2x^2 - 1$
 $y^3 = 2x_1^2 - 1 \rightarrow y_1 = \sqrt[3]{2x_1^2 - 1}$
 $y^3 = 2x_2^2 - 1 \rightarrow y_2 = \sqrt[3]{2x_2^2 - 1}$
 $\Rightarrow 2x_1^2 = 2x_2^2 \Rightarrow \sqrt[3]{2x_1^2 - 1} = \sqrt[3]{2x_2^2 - 1} \Rightarrow y_1 = y_2 \Rightarrow$ تابع است
 (۱)

ب) $y + y = 2x^2 - 1$
 $y(y+1) = 2x_1^2 - 1$
 $y(y+1) = 2x_2^2 - 1$
 $\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2 \Rightarrow y_1(y_1+1) = y_2(y_2+1)$
 باین صفه بخون

الف) تابع است $y = x^2 - 1$
 ب) تابع نیست $|y| + x = x^2 + 1$ if $x=0 \Rightarrow |y|=1 \Rightarrow y = \pm 1 \Rightarrow x=0 \Rightarrow y = \pm 1$
 ج) تابع است $y^3 + x^2 = xy - 1$ if $x=0 \Rightarrow y^3 = -1 \Rightarrow y = -1$
 د) تابع است $\sqrt[3]{y} + \sqrt[3]{x} = 2$ چون فوجی و علامت تغییر نمی‌کند

الف) $\tan \frac{\pi}{4} = \frac{\sin \frac{\pi}{4}}{\cos \frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$
 ب) $\tan \frac{3\pi}{4} = \frac{\sin \frac{3\pi}{4}}{\cos \frac{3\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$
 ج) $\tan \frac{5\pi}{4} = \frac{\sin \frac{5\pi}{4}}{\cos \frac{5\pi}{4}} = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 1$
 د) $\tan \frac{7\pi}{4} = \frac{\sin \frac{7\pi}{4}}{\cos \frac{7\pi}{4}} = \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$

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الف) $\sin \alpha = \frac{1}{2} \Rightarrow \alpha = \left\{ 2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6} \right\}$
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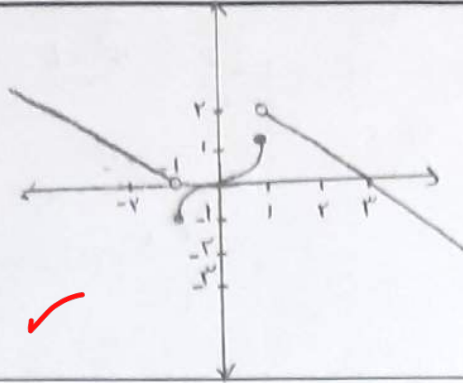
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$$\begin{cases} x > 1 & -x + 3 \\ -1 \leq x \leq 1 & x \\ x < -1 & -x - 1 \end{cases}$$

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الف) $y = (x-2)(x-3)(x-4)$

$$\begin{array}{c|ccc} & 2 & 3 & 4 \\ \hline & - & + & - \\ \hline & \phi & + & \phi & + \end{array} \Rightarrow \begin{cases} y=0: \{2, 3, 4\} \\ y<0: (-\infty, 2) \cup (3, 4) \\ y>0: (2, 3) \cup (4, +\infty) \end{cases}$$

ب) $y = (x-2)(x-3)^2(x-4)$

$$\begin{array}{c|ccc} & 2 & 3 & 4 \\ \hline & + & - & - \\ \hline & \phi & - & \phi & + \end{array} \Rightarrow \begin{cases} y=0: \{2, 3, 4\} \\ y<0: (2, 3) \cup (3, 4) \\ y>0: (-\infty, 2) \cup (4, +\infty) \end{cases}$$

الف) $\frac{(x-2)(x-4)}{x-3}$

$$\begin{array}{c|ccc} & 2 & 3 & 4 \\ \hline & - & + & - \\ \hline & \phi & + & \phi & + \end{array} \Rightarrow \begin{cases} y=0: \{2, 4\} \\ y \rightarrow \text{تعریف نشده}: \{3\} \\ y<0: (-\infty, 2) \cup (3, 4) \\ y>0: (2, 3) \cup (4, +\infty) \end{cases}$$

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ب) $\frac{(x-2)(x-4)}{(x-3)^2}$

$$\begin{array}{c|ccc} & 2 & 3 & 4 \\ \hline & + & - & - \\ \hline & \phi & - & \phi & + \end{array} \Rightarrow \begin{cases} y=0: \{2, 4\} \\ y \rightarrow \text{تعریف نشده}: \{3\} \\ y<0: (2, 3) \cup (3, 4) \\ y>0: (-\infty, 2) \cup (4, +\infty) \end{cases}$$

الف) $y = \frac{x^3 - 3x^2 + 2x}{x-3} = \frac{(x+2)(x-1)(x-2)}{x-3}$

$$\begin{array}{c|ccc} & -2 & 1 & 3 \\ \hline & + & - & - \\ \hline & \phi & - & \phi & + \end{array}$$

$y=0: \{1, 2\}$, $y<0: (2, 3) \cup (3, +\infty)$, $y>0: (-\infty, 2) \cup (3, +\infty)$

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ب) $y = \frac{x^3 - 3x^2 + 2x}{x^2 - 4x + 3} = \frac{(x-1)(x-2)}{(x-1)(x-3)}$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline & + & + & - \\ \hline & \phi & + & \phi & + \end{array}$$

$y=0: \{2\}$, $y<0: (2, 3)$, $y>0: (-\infty, 1) \cup (1, 2) \cup (3, +\infty)$

الف) $\frac{x^4 - x^2}{x^2 + x + 2} = \frac{x^2(x^2 - 1)}{x^2 + x + 2} = \frac{x^2(x-1)(x+1)}{x^2 + x + 2}$

	-1	* 0	+1	
$x^2 - x^2$	+	-	-	+
$-x^2 + x + 2$	+	+	+	+
جواب	+	-	-	+

ب) $\frac{x^2 + 3x + 4}{x^2 + 2x + 3}$

$x^2 + 3x + 4$	+
$x^2 + 2x + 3$	+
حاصل	+

$$y_1^3 + y_1 = 3x_1^2 - 1$$

$$y_2^3 + y_2 = 3x_2^2 - 1 \xrightarrow{x_1 = x_2}$$

$$y_1^3 + y_1 = 3x_1^2 - 1$$

$$y_2^3 + y_2 = 3x_2^2 - 1$$

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$$\rightarrow y_1^3 + y_1 = y_2^3 + y_2 \rightarrow$$

$$\rightarrow (y_1^3 - y_2^3) + (y_1 - y_2) = 0 \rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2) + (y_1 - y_2) = 0$$

$$\rightarrow (y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2 + 1) = 0$$

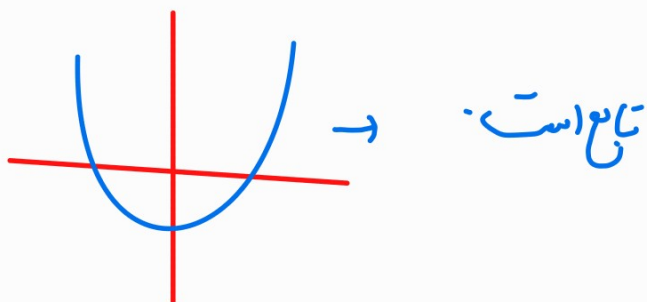
$$y_1 = y_2 \quad \checkmark \quad \text{تابع است} \rightarrow$$

$$y_1^2 + y_2 y_1 + y_2^2 + 1 = 0 \rightarrow \Delta = (y_2)^2 - 4(y_2^2 + 1) = -3y_2^2 + 4 \rightarrow \Delta < 0$$

ریشه ندارد

۲- الف) چون توان ۳ فرد است دلیل مختصه نه بنی رابطه تابع است.

$$y = x^2 - 1$$



تابع است

$$\sqrt[3]{y} + \sqrt[3]{x} = 1$$

تعریف ریاضی ()

$$\sqrt[3]{y_1} + \sqrt[3]{x_1} = 1$$

$$\sqrt[3]{y_2} + \sqrt[3]{x_2} = 1$$

$$\xrightarrow{x_1 = x_2 = x} \sqrt[3]{y_1} + \sqrt[3]{x} = 1$$

$$\sqrt[3]{y_2} + \sqrt[3]{x} = 1 \rightarrow \sqrt[3]{y_1} = \sqrt[3]{y_2} \rightarrow y_1 = y_2$$

تابع است

ج. این رابطه تابع است و نباید مثال نقض بزنیم

تعریف ریاضی

$$y^3 + x^3 = 2x - 1$$

$$y_1^3 + x_1^3 = 2x_1 - 1$$

$$y_2^3 + x_2^3 = 2x_2 - 1$$

$$\xrightarrow{x_1 = x_2} y_1^3 = y_2^3 \rightarrow y_1 = y_2$$

تابع است ..