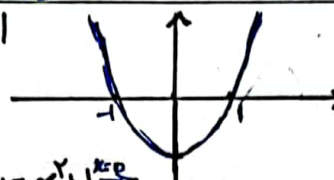


آزمون خط

الف) $y^2 = 2x^2 - 1 \Rightarrow y_1^2 = 2x_1^2 - 1$
 $y_2^2 = 2x_2^2 - 1$
 $x_1 = x_2 \Rightarrow y_1^2 = y_2^2 \Rightarrow y_1 = y_2$ ✓

ب) $y^2 + y = 2x^2 - 1 \Rightarrow y_1^2 + y_1 = 2x_1^2 - 1$
 $y_2^2 + y_2 = 2x_2^2 - 1$
 $x_1 = x_2 \Rightarrow y_1^2 + y_1 = y_2^2 + y_2 \Rightarrow y_1^2 - y_2^2 + y_1 - y_2 = 0 \Rightarrow (y_1 - y_2)(y_1 + y_2 + 1) = 0 \Rightarrow y_1 = y_2$ ✓

① $(y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2 + 1) = 0 \Rightarrow \Delta = y_1^2 - 4y_1 y_2 - 4 = 0 \Rightarrow y_1 = 2y_2 - 4$
 اگر $y_1 = 2y_2 - 4$ را در $y^2 + y = 2x^2 - 1$ قرار دهیم:
 $(2y_2 - 4)^2 + (2y_2 - 4) = 2x^2 - 1 \Rightarrow 4y_2^2 - 14y_2 + 15 = 2x^2 - 1 \Rightarrow 4y_2^2 - 14y_2 + 16 = 2x^2$
 $2y_2^2 - 7y_2 + 8 = x^2$
 این معادله برای x حقیقی بودن باید $\Delta \geq 0$ باشد.
 $\Delta = 49 - 4(2y_2^2 - 7y_2 + 8) \geq 0 \Rightarrow 49 - 8y_2^2 + 28y_2 - 32 \geq 0 \Rightarrow -8y_2^2 + 28y_2 + 17 \geq 0$
 این معادله برای y_2 حقیقی بودن باید $\Delta \geq 0$ باشد.
 $\Delta = 28^2 - 4(-8)(17) = 784 + 544 = 1328 > 0$
 پس $y_1 = y_2$ است. ✓

الف) $y = x^2 - 1$
 تابع است


ج) $y^2 + x^2 = 2x - 1 \Rightarrow y_1^2 + x_1^2 = 2x_1 - 1$
 $y_2^2 + x_2^2 = 2x_2 - 1$
 $x_1 = x_2 \Rightarrow y_1^2 = y_2^2 \Rightarrow y_1 = y_2$
 تابع است ✓

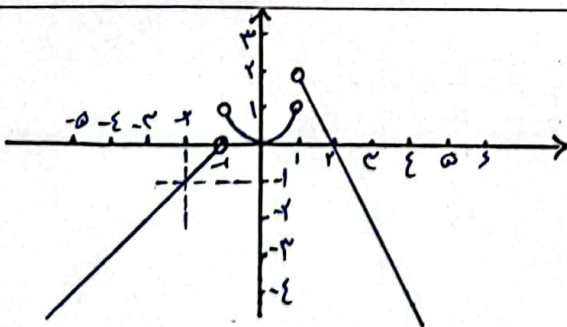
ب) $|y| + x = x^2 + 1 \Rightarrow |y| = x^2 - x + 1$
 $|y| = 1 \Rightarrow y = \pm 1$
 تابع نیست ✓

د) $\sqrt{y} + \sqrt{x} = 2 \Rightarrow \sqrt{y} = 2 - \sqrt{x} \Rightarrow y = (2 - \sqrt{x})^2$
 $\sqrt{y_1} = 2 - \sqrt{x_1}$
 $\sqrt{y_2} = 2 - \sqrt{x_2}$
 $x_1 = x_2 \Rightarrow \sqrt{y_1} = \sqrt{y_2} \Rightarrow y_1 = y_2$
 تابع هست ✓

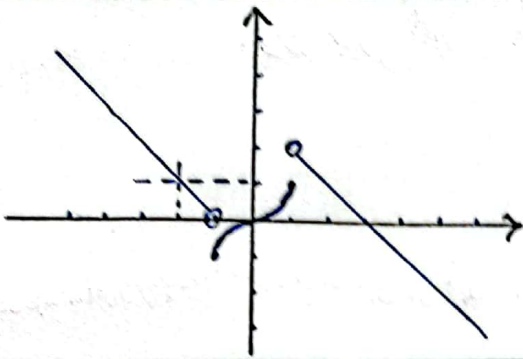
الف) $\tan \frac{\pi}{4} = \frac{\sin \frac{\pi}{4}}{\cos \frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$
 ب) $\cot \frac{\sqrt{\pi}}{5} = \frac{\cos \frac{\sqrt{\pi}}{5}}{\sin \frac{\sqrt{\pi}}{5}} = \frac{-\frac{\sqrt{2}}{2}}{-\frac{1}{2}} = \sqrt{2}$
 ج) $\tan 45^\circ = \frac{\sin 45^\circ}{\cos 45^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$
 د) $\cot 18^\circ = \frac{\cos 18^\circ}{\sin 18^\circ} = \frac{1}{0} = \infty$ (تعریف نشده) ✓

الف) $\sin \alpha = \frac{1}{4} \Rightarrow \alpha = \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$

ب) $\cos \alpha = \frac{\sqrt{2}}{4} \Rightarrow \alpha = \left\{ \frac{\pi}{4}, \frac{7\pi}{4} \right\}$ ✓



✓



Ⓟ ✓

6

الف) $y = (x-1)(x-2)(x-3)$
 $x=1 \quad x=2 \quad x=3$

x	1	2	3	
$f(x)$	-	+	-	+

ب) $y = (x-1)(x-2)(x-3)$ (نفسه)

x	1	2	3	
$f(x)$	+	-	+	-

Ⓟ ✓

7

الف) $y = \frac{(x-1)(x-3)}{x-2}$

x	1	2	3	
$f(x)$	-	+	-	+

ب) $y = \frac{(x-1)(x-3)}{(x-2)^2}$

x	1	2	3	
$f(x)$	+	-	+	-

Ⓟ ✓

8

الف) $y = \frac{x^2 - 2x + 2}{(x-1)^2}$

ب) $y = \frac{x^2 - 2x + 2}{(x-1)^2}$

$\frac{x^2 - 2x + 2}{(x-1)^2} = \frac{x^2 - 2x + 1 + 1}{(x-1)^2} = \frac{(x-1)^2 + 1}{(x-1)^2} = 1 + \frac{1}{(x-1)^2}$

x	1	2	3	
$f(x)$	+	-	+	-

x	1	2	3	
$f(x)$	+	+	-	+

Ⓟ ✓

9

الف) $y = \frac{x^2 - x^2}{x^2 + x + 2} = \frac{0}{x^2 + x + 2}$

ب) $y = \frac{x^2 - x^2}{x^2 + x + 2} = \frac{0}{x^2 + x + 2}$

x	-1	0	1	
$f(x)$	-	+	-	+

x	$-\infty$	$+\infty$
$f(x)$	+	+

Ⓟ ✓

10