

۱۹۷۸ آزمون

الف) $y = \sqrt{\frac{x^3 - x^2}{x^3 - x}}$

$\rightarrow \frac{x^3 - x^2}{x^3 - x} \geq 0 \rightarrow \frac{x^2(x-1)(x+1)}{x(x-1)(x+1)} \geq 0$

$\frac{-1}{+} \frac{0}{+} \frac{1}{+}$

$D_f = \mathbb{R} - \{-1, 0, 1\}$

$D_f = (-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, +\infty)$

الف) $y = \frac{1}{\sqrt{x+|x|}}$

$\rightarrow x+|x| > 0 \xrightarrow{\text{if}} \begin{matrix} x > 0 & \checkmark \\ x = 0 & \times \\ x < 0 & \times \end{matrix}$

$D_f = \mathbb{R}^+ \cup (0, +\infty)$

الف) $y = \frac{1}{\sqrt{2-[x]}}$

$\rightarrow 2-[x] > 0 \rightarrow [x] < 2$

$\Rightarrow D_f = (-\infty, 2)$

الف) $y = \sqrt{[x - \frac{1}{p}] + [x + \frac{1}{p}]}$

$x - \frac{1}{p} = a : \text{عدد صحیح} \rightarrow -x + \frac{1}{p} = -a$

$[a] + [-a] = \begin{cases} 0 & a \in \mathbb{Z} \\ -1 & a \notin \mathbb{Z} \end{cases}$

$x - \frac{1}{p} \in \mathbb{Z} \Leftrightarrow 2 \text{ عدد صحیح باشد}$

$D_f = \{\frac{1}{p} + n \mid n \in \mathbb{Z}\}$

الف) $y = \frac{p}{p \sin^2 x - 1}$

$\rightarrow p \sin^2 x - 1 \neq 0 \rightarrow p \sin^2 x \neq 1$

$\rightarrow \sin^2 x \neq \frac{1}{p} \rightarrow \sin x \neq \pm \sqrt{\frac{1}{p}} \rightarrow \begin{matrix} \frac{\pi}{2}, \frac{3\pi}{2} \\ \frac{\pi}{6}, \frac{5\pi}{6} \\ \frac{7\pi}{6}, \frac{11\pi}{6} \end{matrix}$

$\Rightarrow D_f = \mathbb{R} - \{\frac{k\pi}{p} + \frac{\pi}{p} \mid k \in \mathbb{Z}\}$

$y = \sqrt{\cos x - 1}$ $\rightarrow \cos x - 1 \geq 0$ $\rightarrow \cos x \geq 1$ $x = 2k\pi$ $D_f = \{2k\pi \mid k \in \mathbb{Z}\}$	$y = \sqrt{2 \sin x - 1}$ $\rightarrow 2 \sin x - 1 \geq 0 \rightarrow 2 \sin x \geq 1$ $\sin x \geq \frac{1}{2}$ $\rightarrow \frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$ $\Rightarrow D_f = \left[2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}\right], k \in \mathbb{Z}$	6
$y = \arcsin(x^2 + 3x + 1)$ $\rightarrow -1 \leq x^2 + 3x + 1 \leq 1$ $\textcircled{1} x^2 + 3x + 1 \geq -1 \rightarrow (x+1)(x+2) \geq -1 \rightarrow x \geq -1$ $\textcircled{2} x^2 + 3x + 1 \leq 1 \rightarrow x(x+3) \leq 0$ $\Rightarrow -3 \leq x \leq 0$ $\Rightarrow D_f = [-3, -1] \cup [-1, 0]$	$\cos y = x - 3$ $\rightarrow -1 \leq x - 3 \leq 1$ $\rightarrow 2 \leq x \leq 4$ $\Rightarrow D_f = [-4, -2] \cup [2, 4]$	7
$y = \log(x^3 - x)$ $\rightarrow x^3 - x > 0 \rightarrow x(x+1)(x-1) > 0$ $\Rightarrow D_f = (-1, 0) \cup (1, +\infty)$	$y = \sqrt{2 - \log_2(x-2)}$ $\textcircled{1} x-2 > 0 \rightarrow x > 2$ $\textcircled{2} 2 - \log_2(x-2) \geq 0 \rightarrow \log_2(x-2) \leq 2$ $\rightarrow x-2 \leq 4 \rightarrow x \leq 6$ $\Rightarrow D_f = (2, 6]$	8
$y = \frac{3}{x^2 - 1}$ $\rightarrow x^2 - 1 \neq 0 \rightarrow x \neq \pm 1$ $\Rightarrow D_f = \mathbb{R} - \{0\}$	$y = \frac{3}{x^2 + 1}$ $\rightarrow x^2 + 1 > 0$ $\Rightarrow D_f = \mathbb{R}$	9
$y = \frac{3}{x^2 - 3}$ $\rightarrow x^2 - 3 \neq 0 \rightarrow x \neq \pm \sqrt{3}$ $\Rightarrow D_f = \mathbb{R} - \{\log \sqrt{3}\}$	$y = \frac{3}{x^2 - 2}$ $\rightarrow x^2 - 2 \neq 0 \rightarrow x \neq \pm \sqrt{2}$ $\Rightarrow D_f = \mathbb{R} - \{\frac{1}{\sqrt{2}}\}$	
$y = \left(\frac{x^2 - 2}{x - 5}\right)!$ $\frac{x^2 - 2}{x - 5} \in \mathbb{N} \rightarrow n = 0, 1, 2, \dots$ $x^2 - 2 = (x - 5)n \rightarrow x = \frac{5n + 2}{n - 1}$ $\Rightarrow D_f = \left\{x = \frac{5n + 2}{n - 1} \mid n = 0, 1, 2, \dots\right\}$	$y = (x+1)!$ $x+1 \in \mathbb{N} \rightarrow x = 0, 1, 2, \dots$ $\Rightarrow D_f = \left\{x = \frac{n-1}{5} \mid n = 0, 1, 2, \dots\right\}$	10