

$$y = \sqrt{\frac{x^3 - x^2}{x^3 - x}}$$

$$\rightarrow \frac{x^3 - x^2}{x^3 - x} \geq 0 \rightarrow \frac{x^2(x-1)(x+1)}{x(x+1)(x-1)} \geq 0$$

$\begin{array}{c} + \quad - \quad + \\ -1 \quad 0 \quad 1 \end{array}$

$\begin{array}{c} + \quad + \quad + \quad + \end{array}$

$$D_f = \mathbb{R} - \{-1, 0, 1\}$$

$$= (-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, +\infty)$$

$$y = \sqrt{\frac{x^2 - 4x}{x^2 - x}}$$

$$\rightarrow \frac{x^2 - 4x}{x^2 - x} \geq 0 \rightarrow \frac{x(x-4)}{x(x-1)(x+1)} \geq 0$$

$\begin{array}{c} -1 \quad + \quad 1 \quad 4 \\ - \quad + \quad + \quad - \quad + \end{array}$

$$D_f = (-1, 0) \cup (0, 1) \cup [4, +\infty)$$

$$y = \frac{1}{\sqrt{x|x|}}$$

$$\rightarrow x|x| > 0 \xrightarrow{\text{if}} \begin{array}{l} x > 0 \quad \checkmark \\ x = 0 \quad \times \\ x < 0 \quad \times \end{array}$$

$$D_f = \mathbb{R}^+ \cup (0, +\infty)$$

$$y = \frac{x+1}{\sqrt{x+|x|}}$$

$$\rightarrow x+|x| > 0 \xrightarrow{\text{if}} \begin{array}{l} x > 0 \quad \checkmark \\ x = 0 \quad \times \\ x < 0 \quad \times \end{array}$$

$$D_f = \mathbb{R}^+ \cup (0, +\infty)$$

$$y = \frac{1}{\sqrt{2-[x]}}$$

$$\rightarrow 2-[x] > 0 \rightarrow [x] < 2$$

برای $[x]$

$$\Rightarrow D_f = (-\infty, 2)$$

$$y = \sqrt{2-[x]}$$

$$\rightarrow 2-[x] \geq 0 \rightarrow [x] \leq 2$$

برای $[x]$

$$\Rightarrow D_f = (-\infty, 3)$$

$$y = \sqrt{\left[x - \frac{1}{x}\right] + \left[x + \frac{1}{x}\right]}$$

$$x - \frac{1}{x} = a : \text{مقدار صحیح} \rightarrow -x + \frac{1}{x} = -a$$

$$[a] + [-a] = \begin{cases} 0 & a \in \mathbb{Z} \\ -1 & a \notin \mathbb{Z} \end{cases}$$

$$x - \frac{1}{x} \in \mathbb{Z} \Leftrightarrow \text{چون برت باقی مانده 2 باشد}$$

$$D_f = \left\{ \frac{1}{x} + n \mid n \in \mathbb{Z} \right\}$$

$$y = \frac{x}{x \sin^2 x - 1}$$

$$\rightarrow x \sin^2 x - 1 \neq 0 \rightarrow x \sin^2 x \neq 1$$

$$\rightarrow \sin^2 x \neq \frac{1}{x} \rightarrow \sin x \neq \pm \sqrt{\frac{1}{x}} \rightarrow \begin{array}{l} \text{نقطه } x_0, y_0 \\ x_1, y_1, x_2, y_2 \end{array}$$

$$\Rightarrow D_f = \mathbb{R} - \left\{ \frac{k\pi}{x} + \frac{\pi}{x} \mid k \in \mathbb{Z} \right\}$$

$$y = \sqrt{\left[x - \frac{1}{x}\right] + \left[x + \frac{1}{x}\right]}$$

$$x + \frac{1}{x} = \left(x - \frac{1}{x}\right) + 1 \quad \left\{ \begin{array}{l} [a] + [a+1] = [2a+1] \\ [a] + [a+1] = [2a+1] \end{array} \right.$$

$$\left[x - \frac{1}{x}\right] + \left[x + \frac{1}{x}\right] = 2\left[x - \frac{1}{x}\right] + 1$$

$$2\left[x - \frac{1}{x}\right] + 1 \geq 0 \rightarrow \left[x - \frac{1}{x}\right] \geq 0$$

$$\rightarrow x - \frac{1}{x} \geq 0 \rightarrow x \geq \frac{1}{x} \Rightarrow D_f = \left[\frac{1}{x}, +\infty\right)$$

$$y = \frac{x}{\sin x - 1}$$

$$\rightarrow \sin x - 1 \neq 0$$

$$\rightarrow \sin x \neq 1 \rightarrow x \neq 2k\pi + \frac{\pi}{2} \rightarrow \text{نقطه}$$

$$\Rightarrow D_f = \mathbb{R} - \left\{ 2k\pi + \frac{\pi}{2} \mid k \in \mathbb{Z} \right\}$$

$y = \sqrt{\cos x - 1}$ $\rightarrow \cos x - 1 \geq 0$ $\rightarrow \cos x \geq 1$ $x = 2k\pi$ $D_f = \{2k\pi \mid k \in \mathbb{Z}\}$	$y = \sqrt{2 \sin x - 1}$ $\rightarrow 2 \sin x - 1 \geq 0 \rightarrow 2 \sin x \geq 1$ $\sin x \geq \frac{1}{2}$ $\rightarrow \frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$ $\Rightarrow D_f = \left[2k\pi + \frac{\pi}{6}, 2k\pi + \frac{5\pi}{6}\right], k \in \mathbb{Z}$
$y = \arcsin(x^2 + 3x + 1)$ $\rightarrow -1 \leq x^2 + 3x + 1 \leq 1$ ① $x^2 + 3x + 1 \geq 0 \rightarrow (x+1)(x+2) \geq 0 \rightarrow x \geq -1$ ② $x^2 + 3x \leq 0 \rightarrow x(x+3) \leq 0 \rightarrow -3 \leq x \leq 0$ $\Rightarrow D_f = [-3, -1] \cup [-1, 0]$	$\cos y = x - 3$ $\rightarrow -1 \leq x - 3 \leq 1$ $\rightarrow 2 \leq x \leq 4$ $\Rightarrow D_f = [-4, -2] \cup [2, 4]$
$y = \log(x^3 - x)$ $\rightarrow x^3 - x > 0 \rightarrow x(x+1)(x-1) > 0$ $\Rightarrow D_f = (-1, 0) \cup (1, +\infty)$	$y = \sqrt{2 - \log_2(x-1)}$ ① $x-1 > 0 \rightarrow x > 1$ ② $2 - \log_2(x-1) \geq 0 \rightarrow \log_2(x-1) \leq 2$ $\rightarrow x-1 \leq 4 \rightarrow x \leq 5$ $\Rightarrow D_f = (1, 5]$
$y = \frac{3}{x^2 - 1} \rightarrow x^2 - 1 \neq 0$ $\rightarrow x^2 \neq 1 \rightarrow x \neq \pm 1$ $\Rightarrow D_f = \mathbb{R} - \{0\}$	$y = \frac{3}{x^2 + 1}$ $\rightarrow x^2 + 1 > 0$ $\Rightarrow D_f = \mathbb{R}$
$y = \frac{3}{x^2 - 3} \rightarrow x^2 - 3 \neq 0 \rightarrow x^2 \neq 3$ $\rightarrow x \neq \pm \sqrt{3}$ $\Rightarrow D_f = \mathbb{R} - \{\pm \sqrt{3}\}$	$y = \frac{3}{x^2 - 2} \rightarrow x^2 - 2 \neq 0 \rightarrow x^2 \neq 2$ $\rightarrow x \neq \pm \sqrt{2}$ $\Rightarrow D_f = \mathbb{R} - \{\pm \sqrt{2}\}$
$y = \left(\frac{3x-2}{x-5}\right)!$ $\frac{3x-2}{x-5} = n \rightarrow n = 0, 1, 2, \dots$ $3x-2 = n(x-5) \rightarrow 3x-2 = nx-5n$ $\rightarrow x(3-n) = 2-5n \rightarrow x = \frac{2-5n}{3-n}$ $\Rightarrow D_f = \left\{x = \frac{2-5n}{3-n} \mid n = 0, 1, 2, \dots\right\}$	$y = (x+1)!$ $x+1 = n \rightarrow n = 0, 1, 2, \dots$ $\rightarrow x = \frac{n-1}{1}$ $\Rightarrow D_f = \left\{x = \frac{n-1}{1} \mid n = 0, 1, 2, \dots\right\}$