

$$\text{الف) } \sqrt{\frac{x^2 - 4}{x^3 - x}} = \frac{x(x-2)}{x(x+1)(x-1)} \Rightarrow \frac{1}{x+1} + \frac{1}{x-1} - \frac{1}{x}$$

$$(-1, 0) \cup (0, 1) \cup (2, +\infty)$$

$$\text{ب) } \sqrt{\frac{x^3 - x^2}{x^3 - x}} = \frac{x^2(x-1)}{x(x^2-1)} \Rightarrow \frac{1}{x+1} + \frac{1}{x-1} - \frac{1}{x} \quad P_f = \mathbb{R} - \{-1, 0, 1\}$$

$$\text{الف) } \frac{x+1}{\sqrt{x+1}} = \sqrt{x+1} \Rightarrow x+1 \geq 0 \Rightarrow x \geq -1 \quad P_f = (0, +\infty)$$

$$\text{ب) } \frac{1}{\sqrt{x+1}} \Rightarrow x+1 > 0 \Rightarrow x > -1 \quad P_f = (0, +\infty)$$

$$\text{الف) } y = \sqrt{2-x} \Rightarrow 2-x \geq 0 \Rightarrow x \leq 2 \Rightarrow x < 3 \quad P_f = (-\infty, 3)$$

$$\text{ب) } y = \frac{1}{\sqrt{2-x}} \Rightarrow 2-x > 0 \Rightarrow x < 2 \Rightarrow x < 2 \quad P_f = (-\infty, 2)$$

$$\text{الف) } y = \sqrt{[x-\frac{1}{2}] + [x+\frac{1}{2}]} = [x-\frac{1}{2}] = [x+\frac{1}{2}] - 1 \Rightarrow x \geq \frac{1}{2} \quad P_f = [\frac{1}{2}, +\infty)$$

$$\text{ب) } y = \sqrt{[x-\frac{1}{2}] + [-x+\frac{1}{2}]} = 0 \Rightarrow x \in [\frac{1}{2}, \frac{1}{2}] \quad P_f = \{\frac{1}{2}\}$$

$$[x] + [-x] \geq 0 \Rightarrow x = n + \frac{1}{2} \quad n \in \mathbb{Z} \quad P_f = \{x \mid x = n + \frac{1}{2}, n \in \mathbb{Z}\}$$

$$\text{الف) } y = \frac{1}{\sin x - 1} \Rightarrow \sin x - 1 \neq 0 \Rightarrow \sin x \neq 1 \Rightarrow x \neq 2k\pi + \frac{\pi}{2}$$

$$P_f = \mathbb{R} - \{2k\pi + \frac{\pi}{2} \mid k \in \mathbb{Z}\}$$

$$\text{ب) } y = \frac{1}{1 - \sin^2 x} \Rightarrow 1 - \sin^2 x \neq 0 \Rightarrow \sin^2 x \neq 1 \Rightarrow \sin x \neq \pm 1 \Rightarrow x \neq 2k\pi + \frac{\pi}{2} \text{ and } x \neq 2k\pi - \frac{\pi}{2}$$

$$\text{ج) } y = \sqrt{x \sin x - 1} \Rightarrow x \sin x \geq \frac{1}{y} \quad D_f = \left\{ x \mid x \in \mathbb{R}, \frac{1}{y} \leq x \sin x \leq \frac{1}{y} \right\}$$

$$\text{ب) } y = \sqrt{\cos x - 1} \Rightarrow x \cos x \geq \frac{1}{y} \quad D_f = \{x \mid x \in \mathbb{R}\}$$

$$\text{ج) } \cos y = |x| - 3 \Rightarrow -1 \leq \cos y \leq 1 \Rightarrow -1 \leq |x| - 3 \leq 1 \Rightarrow 2 \leq |x| \leq 4$$

$$D_f = [-4, -2] \cup [2, 4]$$

$$\text{ب) } y = \arcsin(x^2 + 3x + 1) \Rightarrow -1 \leq x^2 + 3x + 1 \leq 1 \Rightarrow x^2 + 3x \leq 0 \Rightarrow x(x+3) \leq 0 \Rightarrow -3 \leq x \leq 0$$

$$D_f = [-3, 0]$$

$$\text{ج) } \sqrt{y - \log_p(x-1)} \Rightarrow y - \log_p(x-1) \geq 0 \Rightarrow y \geq \log_p(x-1) \Rightarrow \log_p(x-1) \leq y \Rightarrow x-1 \leq p^y \Rightarrow x \leq 1 + p^y$$

$$\text{ب) } y = \log(x^2 - x) \Rightarrow x^2 - x > 0 \Rightarrow x(x-1) > 0 \Rightarrow x < 0 \text{ or } x > 1$$

$$D_f = (-\infty, 0) \cup (1, \infty)$$

$$\text{ج) } y = \frac{x}{x^2 + 1} \Rightarrow x \neq -1 \quad D_f = \mathbb{R}$$

$$\text{ب) } y = \frac{x}{x^2 - 1} \Rightarrow x \neq 1 \text{ or } x \neq -1 \quad D_f = \mathbb{R} - \{-1, 1\}$$

$$\text{ج) } y = \frac{x}{x^2 - 2} \Rightarrow x \neq \sqrt{2} \text{ or } x \neq -\sqrt{2} \quad D_f = \mathbb{R} - \{\pm\sqrt{2}\}$$

$$\text{ب) } y = \frac{x}{x^2 - 3} \Rightarrow x \neq \sqrt{3} \text{ or } x \neq -\sqrt{3} \quad D_f = \mathbb{R} - \{\pm\sqrt{3}\}$$

$$\text{ج) } y = (x+1)! \quad x+1 = k \quad k \in \mathbb{N} \quad x = k-1 \quad D_f = \{x \mid x = k-1, k \in \mathbb{N}\}$$

$$\text{ب) } \left( \frac{x-1}{x-2} \right)! \quad \frac{x-1}{x-2} = k \quad k \in \mathbb{N} \quad x = \frac{2k+1}{k-1}$$

$$D_f = \left\{ x \mid x = \frac{2k+1}{k-1}, k \in \mathbb{N}, k \neq 1 \right\}$$