

$y = \sqrt{x} f(x)$

x	-1	$-1/2$	0	$1/2$	1
y	0	-1	0	-1	0

$1 \rightarrow 0$
 $1/2 \rightarrow 0$
 $0 \rightarrow -1$
 $-1/2 \rightarrow 0$

$Df = \begin{bmatrix} -1/2 & 0 \end{bmatrix} \cup \begin{bmatrix} 1/2 & 0 \end{bmatrix}$

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$y = \sqrt{\frac{-x}{f(x)}}$

x	-1	0	1	2	3
y	0	1	0	1	0

$1 \rightarrow 0$
 $2 \rightarrow 0$
 $0 \rightarrow 0$
 $-1 \rightarrow 0$

$Df = \begin{pmatrix} -1/2 & 0 \\ 0 & 1 \end{pmatrix} \cup \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cup \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow$ عدد صحیح

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$f(x) - 2f(x) = x^2 - 2x + 2$

$f(-2) = 2$

$x=2 \Rightarrow f(2) - 2f(2) = 2^2 - 2 \cdot 2 + 2 \Rightarrow -f(2) = 0 \Rightarrow f(2) = 0$

$x=-2 \Rightarrow f(-2) - 12 = 2^2 - 2 \cdot (-2) + 2 = 12 \Rightarrow f(-2) = 12 + 12 = 24$

$f(x) - 2f(x) = 2 \Rightarrow f(x) = -2 \Rightarrow f(x) = x^2 - 2x$

۳- ابتدا $f(x)$ را بیابیم:

$f(x) = \begin{cases} x - \sqrt{x+2} & x > 1 \\ 2x+3 & x \leq 1 \end{cases}$

$f(1) = 1 - \sqrt{1+2} = 1 - \sqrt{3}$

$f(f(1)) \Rightarrow f(1) = 2 + 3 = 5 \Rightarrow f(5) = 5 - \sqrt{5+2} = 5 - \sqrt{7}$

$f(f(5)) \Rightarrow f(5) = 5 - \sqrt{7} = 2 \Rightarrow f(2) = 2 + 3 = 5$

$f(f(5)) + f(f(1)) = 2 + 5 = 7$

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$f(x) = ax^2 - bx + 2$

$f(x-1) - f(x) = 4x + 2$

$a - b = -2 - a = -1$

$a(x-1) - b(x-1) + 2 - (ax^2 - bx + 2) = 4x + 2$

$ax^2 - 1 - bx + b + 2 - ax^2 + bx - 2 = 4x + 2$

$-bx + b = 4x + 2$

$-b = 4 \Rightarrow b = -4$

$a + b = 2 \Rightarrow a - 4 = 2 \Rightarrow a = 6$

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$$f(n) = n^2 - 2n + 1 \quad 2f(2) \Rightarrow f(2) = 9 - 4 + 1 = 1 \Rightarrow 2f(2) = 2$$

$$2f(2) + f(-1) = \frac{2 \times 1}{1} = \frac{2}{1} = 2$$

$$2f(2) \Rightarrow f(2) = 2 - 4 + 1 = -1 \Rightarrow f(2) = 1$$

$$f(n) = \begin{cases} 2n+1 & n \geq 2 \\ n^2 & n < 2 \end{cases} \quad 2f(2) \Rightarrow f(2) = 1 \times 2 + 1 = 3 \Rightarrow 2f(2) = 6$$

$$2f(-2) \Rightarrow f(-2) = 4 \Rightarrow 2f(-2) = 8$$

$$\frac{2f(2) - 2f(-2)}{f(2)} = \frac{6 - 8}{3} = \frac{-2}{3} = -\frac{2}{3}$$

$$f(n) = \{(2, 4), (3, 2), (4, 1), (1, 4)\} \quad f(f(2)) \Rightarrow f(4) = 1 \Rightarrow f(1) = 4$$

$$\frac{f(f(2)) - f(f(3))}{f(1) - 2} = \frac{4 - 2}{4 - 2} = \frac{2}{2} = 1$$

$$\rightarrow 0$$

$$f(n) = \frac{n^2 + 2n + 9}{n^2 + n + 4}$$

$$f(f(\sqrt{2}-1)) = \frac{1}{14} f(\sqrt{2}-1) = \frac{(\sqrt{2}-1)^2 + 2(\sqrt{2}-1) + 9}{(\sqrt{2}-1)^2 + (\sqrt{2}-1) + 4} = \frac{2 + 2\sqrt{2} - 1 + 2\sqrt{2} - 2 + 9}{2 + 2\sqrt{2} - 1 + \sqrt{2} - 1 + 4} = \frac{8 + 4\sqrt{2}}{6 + 3\sqrt{2}} = \frac{2(4 + 2\sqrt{2})}{3(2 + \sqrt{2})} = \frac{2(2 + \sqrt{2})}{3} = \frac{4 + 2\sqrt{2}}{3}$$

$$f(2) = \frac{4 + 4 + 9}{4 + 2 + 4} = \frac{17}{10} = \frac{1}{10}$$

$$\frac{1}{10} + 2 = \frac{1}{10} + \frac{20}{10} = \frac{21}{10}$$

$$= \frac{21}{10} = 2.1$$

$$f(n) = n^2 + \frac{1}{n^2}$$

$$(f(\sqrt{n}))^2 - f(n) = n^2 + \frac{1}{n^2} + 2 - n^2 - \frac{1}{n^2} = 2$$

$$(f(\sqrt{n}))^2 = \frac{1}{n} + n \Rightarrow f(\sqrt{n})^2 = n^2 + \frac{1}{n^2} + 2, f(n) = n^2 + \frac{1}{n^2}$$