

$f(n) = n^2 - 2n + 1$ $2f(2) \Rightarrow f(2) = 9 - 4 + 1 = 1 \Rightarrow 2f(2) = 2$ $2f(2) + f(-1) = \frac{2 \times 1}{1} = \frac{2}{1} = 2$ $f(-1) \Rightarrow f(-1) = 1 + 2 + 1 = 4 \Rightarrow f(-1) = 4$ $f(2) \Rightarrow f(2) = 4 - 4 + 1 = 1 \Rightarrow f(2) = 1$	6
$f(n) = \begin{cases} 2n+1 & n \geq 2 \\ n^2 & n < 2 \end{cases}$ $2f(2) \Rightarrow f(2) = 1 \times 2 + 1 = 3 \Rightarrow 2f(2) = 6$ $2f(-2) \Rightarrow f(-2) = 4 \Rightarrow 2f(-2) = 8$ $f(2) = \frac{2 \times 4 + 1}{2} = \frac{9}{2}$ $\frac{2f(2) - 2f(-2)}{f(2)} = \frac{6 - 8}{\frac{9}{2}} = \frac{-2}{\frac{9}{2}} = -\frac{4}{9}$	7
$f(n) = \{(2,4), (3,2), (4,1), (1,4)\}$ $f(f(2)) \Rightarrow f(4) = 1 \Rightarrow f(1) = 4$ $f(f(3)) \Rightarrow f(2) = 4 \Rightarrow f(4) = 1$ $\frac{f(f(2)) - f(f(3))}{f(1) - 2} = \frac{1 - 4}{4 - 2} = -\frac{3}{2}$ $\boxed{\rightarrow 0}$	8
$f(n) = \frac{n^2 + 2n + 9}{n^2 + n + 4}$ $f(f(\sqrt{2}-1)) = \frac{1}{14}$ $f(2) = \frac{4 + 4 + 9}{4 + 2 + 4} = \frac{17}{10}$ $f(\sqrt{2}-1) = \frac{(\sqrt{2}-1)^2 + 2(\sqrt{2}-1) + 9}{(\sqrt{2}-1)^2 + (\sqrt{2}-1) + 4} = \frac{2 + 2\sqrt{2} - 1 + 2\sqrt{2} - 2 + 9}{2 + 2\sqrt{2} - 1 + \sqrt{2} - 1 + 4} = \frac{8 + 4\sqrt{2}}{6 + 3\sqrt{2}} = \frac{2 + 4\sqrt{2}}{3 + 3\sqrt{2}} = \frac{2 + 4\sqrt{2}}{3(1 + \sqrt{2})} = \frac{2 + 4\sqrt{2}}{3(1 + \sqrt{2})} \cdot \frac{1 - \sqrt{2}}{1 - \sqrt{2}} = \frac{2 - 2\sqrt{2} + 4 - 4\sqrt{2}}{3(1 - 2)} = \frac{6 - 6\sqrt{2}}{-3} = 2\sqrt{2} - 2$	9
$f(n) = n^2 + \frac{1}{n^2}$ $(f(\sqrt{n}))^2 - f(n) = n^2 + \frac{1}{n^2} + 2 - n^2 - \frac{1}{n^2} = 2$ $(f(\sqrt{n}))^2 = \frac{1}{n} + n \Rightarrow f(\sqrt{n})^2 = n^2 + \frac{1}{n^2} + 2, f(n) = n^2 + \frac{1}{n^2}$	10