

1

$f(x):$

$y = \sqrt{x f(x)}$

$\Rightarrow x f(x) \geq 0 \rightarrow$ علامت x و $f(x)$ باید یکسان باشد

$\Rightarrow Df = [-\infty, 0] \cup [2, \infty]$

2

$f(x):$

$y = \sqrt{\frac{-x}{f(x)}}$

$\Rightarrow \frac{-x}{f(x)} \geq 0, f(x) \neq 0 \rightarrow$ علامت x و $f(x)$ باید مختلف باشد

$\Rightarrow Df = (0, 2) \cup (-2, 0)$

$\rightarrow$ عدد صحیح $1, -1, -2 \rightarrow$

3

$f(x) - 2f(2) = x^2 - 2x + 4$

$f(2) - 2f(2) = 4 - 4 + 4 \Rightarrow -f(2) = 2 \Rightarrow f(2) = -2$

$f(-2) - 2f(-2) = (-2)^2 - 2(-2) + 4 \Rightarrow f(-2) + 4 = 8 \Rightarrow f(-2) = 4$

4

$f(x) = \begin{cases} x - \sqrt{x+4} & x > 2 \\ 2x+2 & x \leq 2 \end{cases}$

$\Rightarrow f(f(\infty)) + f(f(1)) = f(\infty - \sqrt{9}) + f(2 \times 1 + 2)$

$= f(7) + f(4) = (2 \times 2 + 2) + (\infty - \sqrt{\infty+4}) = 9$

5

$f(x) = ax^2 - bx + 2$

$f(x-1) - f(x) = 4x + 2$

$(a(x-1)^2 - b(x-1) + 2) - (ax^2 - bx + 2) = 4x + 2$

$\Rightarrow (ax^2 - 2ax + a - bx + b + 2) - (ax^2 - bx + 2) = 4x + 2$

$\Rightarrow -2ax + a + b = 4x + 2 \xrightarrow{x=1} -2a + a + b = 4 + 2 \Rightarrow a - b = -6$

$$f(x) = x^2 - 3x + 1$$

$$\frac{f(2) + f(-1)}{f'(2)} = \frac{2(2^2 - 3 \times 2 + 1) + ((-1)^2 - 3(-1) + 1)}{(2^2 - 3 \times 2 + 1)^2} = \frac{2 + 5}{(-1)^2} = 7$$

6

$$f(x) = \begin{cases} 2x+1 & x \geq 2 \\ x^2 & x < 2 \end{cases}$$

$$\frac{f(2) - f(-2)}{f'(2)} = \frac{2(2 \times 2 + 1) - 2((-2)^2)}{2 \times 2 + 1} = \frac{29 - 8}{5} = \frac{21}{5}$$

7

$$f = \{(2, 4), (4, 1), (1, 4)\}$$

$$\frac{f(f(2)) - f(f(4))}{f(2) - 2} = \frac{f(4) - f(1)}{f(2) - 2} = \frac{4 - 4}{4 - 2} = \frac{0}{2} = 0$$

8

$$f(x) = \frac{x^2 + 5x + 9}{x^2 + 5x + 2}$$

$$f(f(\sqrt{2}-1)) = f\left(\frac{\sqrt{2}-1+5\sqrt{2}-5+9}{\sqrt{2}-1+5\sqrt{2}-5+2}\right) = f\left(\frac{1}{1}\right) = \frac{1^2 + 5 \times 1 + 9}{1^2 + 5 \times 1 + 2} = \frac{14}{8} = \frac{7}{4}$$

9

$$f(x) = x^2 + \frac{1}{x^2}$$

$$(f(\sqrt{x}))^2 - f(x) = \left(\left(\sqrt{x}\right)^2 + \frac{1}{(\sqrt{x})^2}\right)^2 - \left(x^2 + \frac{1}{x^2}\right) = x^2 + \frac{1}{x^2} + 2 - \left(x^2 + \frac{1}{x^2}\right) = 2$$

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