

$$y = \sqrt{x f(x)} \Rightarrow x f(x) \geq 0 \Rightarrow \begin{cases} x \leq 0, f(x) \leq 0 \\ x \geq 0, f(x) \geq 0 \end{cases} \quad D_f = [-\infty, 0] \cup [1, \infty]$$

(۲) ✓

$$y = \sqrt{\frac{-x}{f(x)}} \Rightarrow \frac{-x}{f(x)} \geq 0 \Rightarrow \begin{cases} x \leq 0, f(x) > 0 \\ x \geq 0, f(x) < 0 \end{cases} \quad D_f = (-2, 0) \cup (0, 2)$$

(۲) ✓

$$f(x) - 2f(2) = x^2 - 3x + 4 \xrightarrow{x=2} f(-2) - 2f(2) = 4 - (-4) + 4 = 12$$

$$\Downarrow x=2$$

$$f(2) - 2f(2) = 4 - 4 + 4 = 4 \Rightarrow f(2) = -2$$

$$\Rightarrow f(-2) = 1.$$

(۲) ✓

$$f(x) = \begin{cases} x - \sqrt{x+4} & x > 3 \\ 2x+3 & x \leq 3 \end{cases} \quad f(f(5)) + f(f(1)) = 9$$

(۲) ✓

$$f(x) = ax^2 - bx + 2, \quad f(x-1) - f(x) = 7x+2$$

$$\xrightarrow{x=1} f(1) - f(0) = 7+2=9 \Rightarrow 1 - (a-b+2) = 9$$

$$\Rightarrow a-b = -10$$

(۲) ✓

$$f(x) = x^r - rx + 1$$

$$\frac{rf(r) + f(-1)}{f^r(r)} = \frac{r(1 - r + 1) + (1 + r + 1)}{(1 - r + 1)^r} = \frac{r + 1}{1} = r$$

Ⓟ ✓

$$f(x) = \begin{cases} rx + 1 & x \geq r \\ x^r & x < r \end{cases}$$

$$\frac{rf(r) - rf(-r)}{f(r)}$$

$$\frac{r(r+1) - r(-r)}{r(r)+1} = \frac{r}{r}$$

Ⓟ ✓

$$f = \{(r, r), (r, r), (1, 1), (1, r)\}$$

$$\frac{f(f(r)) - f(f(1))}{f(1) - r} = \frac{r - r}{1 - r} = 0$$

Ⓟ ✓

$$f(x) = \frac{x^r + rx + 1}{x^r + rx + 1} = \frac{(x+r)^r + 1}{(x+r)^r + 1}$$

$$f(\sqrt{r}-r) = \frac{(\sqrt{r})^r + 1}{(\sqrt{r})^r + 1} = \frac{1}{1} = 1$$

Ⓟ ✓

$$f(x) = x^r + \frac{1}{x^r}$$

$$(f(\sqrt{x}))^r - f(x) = x^r + \frac{1}{x^r} + r - x^r - \frac{1}{x^r} = r$$

Ⓟ ✓