رسا طالبی

۱) $a_n = \frac{1}{2}, 1, 2, \ldots \Rightarrow a_n = a_1 q^{n-1} = \frac{1}{2}(2)^{n-1}$

الف)

$a_{10} = \frac{1}{2}(2)^{10-1} = 256 \qquad \frac{a_2}{a_1} = q = \frac{1}{\frac{1}{2}} = 2$

ب) $\frac{a_{17}}{a_{13}} = \frac{a_{13} q^4}{a_{13}} = q^4 = 2^4 = 16$

ج) $b^2 = ac = a_4 a_{10} = a_1^2 q^{12} = (a_1 q^6)^2 = (a_7)^2$

$a_7 = a_1 q^6 = \frac{1}{2} \times 2^6 = 2^5 = 32$

واسطه هندسی

د) $a_n = 128 = a_1 (q)^{n-1} = \frac{1}{2}(2)^{n-1} = 2^7 = 2^{-1} \times 2^{n-1} \Rightarrow$

$n - 1 = 8 \Rightarrow \boxed{n = 9}$

۲) $\frac{a_8}{a_5} = q^3 = \frac{96}{12} = 8 \Rightarrow \boxed{q = 2} \Rightarrow$

$a_{10} = a_8 q^2 = 96 \times (2^2) = 384$

۳) الف) $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = (\text{وسط جمله})^n = a_3^n = a_3^5 = 3^5$

$\Rightarrow \boxed{a_3 = 3}$

ب) $a_3^n = (a_1 \cdot a_n)^{\frac{n}{2}} = 3^5 = (a_1 \cdot a_5)^{\frac{5}{2}} \Rightarrow a_1 \cdot a_5 = 3^2 = 9$

حاصل ضرب جملات

۴) $2^a \cdot 2^b = (4\sqrt{2})^2 = 32 = 2^{a+b} \Rightarrow a + b = 5$

b و a واسطه حسابی $\Rightarrow a = b + 2d \Rightarrow b + b + 2d = 2b + 2d = 5$

واسطه حسابی $b + d = \frac{5}{2}$ یا راه دوم: واسطه $= \frac{a+b}{2} = \frac{5}{2}$

۵) $(1+x)(1-x) = x^2$ (سوال ۵)

$1 - x^2 = x^2$

$1 = 2x^2$

$x^2 = \frac{1}{2}$

$x = \pm \frac{1}{\sqrt{2}}$

۶)

$$\left.\begin{array}{l} a_1 + a_4 = 28 \\ a_2 + a_3 = 12 \end{array}\right\} a_1 + a_2 + a_3 + a_4 = 40$$

$$\frac{a(q^n - 1)}{q - 1} = 40$$

$$\frac{a_1 + a_4}{a_2 + a_3} = \frac{7}{3} = \frac{\cancel{a_1}(1 + q^3)}{\cancel{a_1}(q + q^2)} = \frac{7}{3} \Rightarrow 3 + 3q^3 = 7q + 7q^2$$

$$3(1 + q^3) = 7q(1 + q)$$

$$3\cancel{(1+q)}(1 + q^2 - q) = 7q\cancel{(1+q)}$$

$$3 + 3q^2 - 3q = 7q \Rightarrow$$

$$3q^2 + 3 - 10q = 0$$

روش روسی

$$\Rightarrow q^2 - 10q + 9 = 0 \Rightarrow$$

$$(q - 9)(q - 1) = 0 \Rightarrow$$

$$q = \frac{9}{3} = 3$$

یا

$$q = \frac{1}{3}$$

$$a_1(1 + q^3) = 28$$

$q = 3$: $a_1(28) = 28 \Rightarrow a_1 = 1 \Rightarrow \boxed{a_4 = 27}$ بزرگترین جمله

$q = \frac{1}{3}$: $a_1\left(\frac{28}{27}\right) = 28 \Rightarrow \boxed{a_1 = 27}$ بزرگترین جمله

$$\boxed{a_4 = 1}$$

۷)

$$a_1 + a_2 + a_3 = 13$$

$$a_1 \times a_2 \times a_3 = (a_2)^3 = 27 = 3^3 \Rightarrow a_2 = 3$$

$$\Rightarrow a_1 + 3 + a_1 q^2 = 13 \Rightarrow a_1(1 + q^2) = 10$$

$$\frac{a_2}{q} = \frac{3}{q} \Rightarrow \frac{10q}{3} = 1 + q^2 \Rightarrow$$

$$3 + 3q^2 - 10q = 0$$

روش روسی

$$\Rightarrow q^2 - 10q + 9 = 0 \Rightarrow$$

$$(q - 9)(q - 1) = 0 \Rightarrow$$

$$q = \frac{9}{3} = 3$$

یا

$$q = \frac{1}{3}$$

$$\frac{3}{3} = \frac{3}{q} = \frac{a_2}{q}$$

$$a_1 = 1,\quad a_2 = 3,\quad a_3 = 9$$

دارای ۲ نوع جواب

$$\frac{3}{\frac{1}{3}} = \frac{3}{q} = \frac{a_2}{q}$$

$$a_1 = 9,\quad a_2 = 3,\quad a_3 = 1$$

۸)

$a_n = ۲, ۶, ۱۸, \dots$

الف) $q = \frac{a_2}{a_1} = ۳ \Rightarrow S_{۱۰} = \frac{a(q^n - ۱)}{q - ۱} = \frac{۲(۳^{۱۰} - ۱)}{۲} = ۳^{۱۰} - ۱$

ب) $P = (a_1 \cdot a_{۱۰})^{\frac{n}{۲}} = (a_1 \cdot \underbrace{a_{۱۰}}_{۲,۳^۹})^۵ = (۲ \times ۲ \times ۳^۹)^۵$

$= (۲^۲ \times ۳^۹)^۵ = ۲^{۱۰} \times ۳^{۴۵}$

۹)

$a_1, a_2, a_3, \dots, a_n$

$a, aq^1, aq^2, \dots, aq^{n-1}$

$aq - a, \; aq^2 - aq, \; \dots, \; aq^{n-1} - aq^{n-2}$

$a(q-1), \; a(q^2 - q), \; \dots, \; a(q^{n-1} - q^{n-2})$

$\times q \quad \times q \quad \times q$

قدر نسبت $= q$

۱۰)

الف) $\boxed{S_n = \frac{n}{۲}(۲a_1 + (n-1)d)}$

$a_1 + a_2 + \dots + a_n = \underbrace{a_1 + a_1 + d + a_1 + ۲d + \dots + a_1 + (n-1)d}_{a_1 \,(n) \,,\; 1+2+3+\dots+n-1 \,(d)}$

$\Rightarrow S_n = n(a_1) + \frac{(n-1)nd}{۲} = \frac{n}{۲}(۲a_1 + (n-1)d)$

ب) $\boxed{S_n = a\left(\frac{q^n - 1}{q - 1}\right)}$

ضرب در q

$S_n = a_1 + a_1 q + a_1 q^2 + \dots + a_1 q^{n-1}$

$S_n q = a_1 q + a_1 q^2 + a_1 q^3 + \dots + a_1 q^n \Rightarrow S_n q - S_n = a_1 q^n - a_1$

$S_n(q - 1) = a_1(q^n - 1)$

$\Rightarrow S_n = \frac{a_1(q^n - 1)}{q - 1}$