

الف) $a_1 = \frac{1}{r}$ $q = \frac{a_2}{a_1} = \frac{1}{\frac{1}{r}} = r$
 $a_{10} = a_1 q^9 = 254$ ✓

ج) $a_2 = 4$ $a_{10} = 254$ ✓
 واسطه = $254 \times 4 = 1016 = \sqrt{1016} = 32$ ✓

ب) $\frac{a_{14}}{a_{13}} = \frac{a_1 q^{14}}{a_1 q^{13}} = q^1 = r = 14$ ✓

$\frac{1}{r} \times r^{n-1} = 128$
 $r^{n-1} = 128 \times r$
 $n-1 = 8$
 $n = 9$ ✓

$a_4 = 12$
 $a_8 = 94$
 $\frac{a_8}{a_4} = \frac{a_1 q^7}{a_1 q^3} \cdot q^4 = \frac{94}{12} = 8$
 $q = 2$

$a_{10} = a_4 \times q^{10-4} = 12 \times 2^6 = 384$ ✓

a_1, a_2, a_3, a_4, a_5
 $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = a_1^5 q^{10}$
 $= 243$
 $\Rightarrow a_1 = a_1 q^4$
 $a_5 a_1^4 = (a_1 q^4)^5 = a_1^5 q^{20} = 243$
 $a_1 = \sqrt[5]{243} \Rightarrow 3$ ✓

$a_1 \times a_5 = a_1 \times a_1 q^4 = a_1^2 q^4 = (a_1 q^2)^2$
 $a_1 \times a_5 = 3^2 = 9$ ✓

$r\sqrt{r} = r^2 \times \frac{1}{r} = r^{\frac{3}{2}}$
 $r^a, r^{\frac{a}{r}}, r^b$
 $(r^{\frac{a}{r}})^r = r^a = r^a \times r^b$
 $a + b = a$

واسطه حسابی = $\frac{a+b}{r} = \frac{a}{r}$ ✓

$2x = 1-x$ $a_7 = x$ $a_{11} = x+1$

س جمله ۳ تا ۷ و جمله ۴ قدم اختلاف در بین ۷ و ۱۱ هم ۴

$x = (1-x)q^4$ $(x+1) = xq^4$
 $q^4 = \frac{x}{1-x} = \frac{x+1}{x} \Rightarrow x^2 = (1-x)(x+1) = 1-x^2$

$2x^2 = 1 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \frac{\pm 1}{\sqrt{2}}$ → (به خاطر q) متقی نمی تواند باشد

$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ✓

$$aq, aq^2, aq^3, aq^4$$

$$aq + aq^2 = 12$$

$$a + aq^3 = 18$$

$$aq(1+q) = 12 \Rightarrow a = \frac{12}{q(1+q)}$$

$$\frac{12}{q(1+q)} (1+q^3) = 18$$

$$12(1-q+q^2) = 18q$$

$$4q^2 - 10q + 3 = 0$$

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{10 \pm \sqrt{44}}{4} = 3 \pm \frac{1}{2} \Rightarrow q$$

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نیز قدرین جمله (27)

دنباله = 1, 3, 9, 27

$$a + aq + aq^2 = 13$$

$$a \times aq \times aq^2 = 27$$

$$(aq)^3 = 27 \Rightarrow aq = 3$$

$$a = \frac{3}{q}$$

$$\frac{3}{q} (1+q+q^2) = 13$$

$$3(1+q+q^2) = 13q$$

$$13q = 3 + 3q + 3q^2$$

$$4q^2 - 10q + 3 = 0$$

$$q = \frac{10 \pm \sqrt{44}}{4} = q = 3 \pm \frac{1}{2} \Rightarrow q = \frac{1}{3}$$

جملات اول تا سوم = (1, 3, 9) و (9, 3, 1) ✓

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الف) $\frac{2, 4, 18}{\times 3 \times 3}$

$$a_1 = 2$$

$$q = 3$$

$$S_{10} = 2 \left(\frac{3^{10} - 1}{3 - 1} \right) = 3^{10} - 1 \quad \checkmark$$

$$= 59048$$

ب) $a_1 \times a_2 \times a_3 \times a_4 \times \dots \times a_{10} = \frac{n(n-1)}{2} a$
 $a \times aq \times aq^2 \times \dots \times aq^9 = a^{10} \times q^{45}$
 $= 2^{10} \times 3^{45} \quad \checkmark$

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دنباله اولیه

$$a, aq, aq^2, aq^3, \dots$$

دنباله ثانویه

$$a - aq, aq - aq^2, \dots$$

$$= a(q-1), aq(q-1), aq^2(q-1) \quad \checkmark$$

در یک دنباله هندسی $\rightarrow \frac{a_2}{a_1} = q$

$$\frac{a_2}{a_1} = \frac{aq(q-1)}{a(q-1)} = q \quad \checkmark$$

بنابراین دنباله هندسی است

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$$S_n = a + (a+d) + (a+2d) + \dots + (a+(n-1)d)$$

$$S_n = (a_1 + (n-1)d) + (a_1 + (n-2)d) + \dots + a_1$$

$$a_1 + (a_1 + (n-1)d) = 2a_1 + (n-1)d$$

$$a_1 + d + (a_1 + (n-2)d) = 2a_1 + (n-1)d$$

$$2S_n = n(2a_1 + (n-1)d) \quad \checkmark$$

$$S_n = \frac{n}{2} (2a_1 + (n-1)d)$$

$$S_n = a + aq + aq^2 + \dots + aq^{n-1}$$

$$aq S_n = aq + aq^2 + aq^3 + \dots + aq^{n-1} + aq^n$$

$$S_n = a + aq + aq^2 + \dots + aq^{n-1}$$

$$aq S_n = aq + aq^2 + \dots + aq^{n-1} + aq^n$$

$$S_n - aq S_n = a - aq^n$$

$$S_n(1-a) = a(1-a^n) \Rightarrow S_n = a \left(\frac{1-a^n}{1-a} \right)$$

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