۱

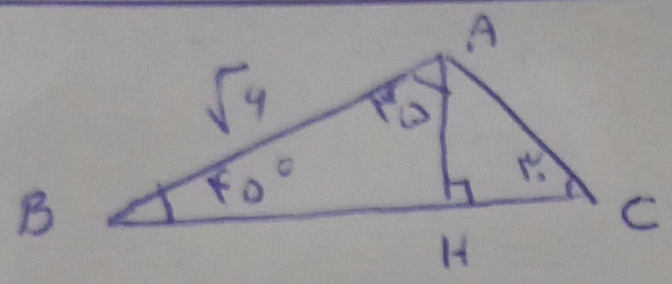
الف) $\sin^2 135 - \cos^2 150 = \frac{2}{4} - \frac{3}{4} = -\frac{1}{4}$

ب) $\frac{1+3\times\frac{1}{3}}{4\times\frac{3}{4}+2\times\frac{1}{2}} = \frac{2}{4} = \frac{1}{2}$

$\cos(\alpha+\alpha) = -\cos\alpha$
$\tan(\alpha+\alpha) = \tan\alpha$

۲

الف)



مثلث AHB قائم‌الزاویه

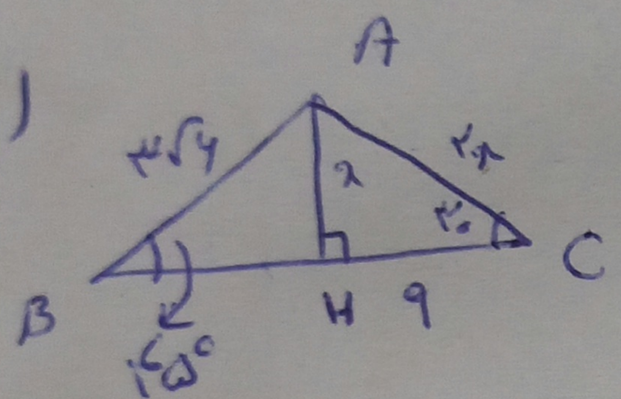
$AH = \sqrt{3}$

در مثلث AHC: $AC^2 = AH^2 + HC^2$

$(2\sqrt{3})^2 = (\sqrt{3})^2 + HC^2$

$HC = 3$

ب)



در مثلث AHC: $AC^2 = AH^2 + HC^2$

$(2x)^2 = x^2 + 81$

$4x^2 = x^2 + 81 \Rightarrow 3x^2 = 81 \Rightarrow x^2 = 27$

$x = 3\sqrt{3}$

در مثلث ABH: $(3\sqrt{6})^2 = (3\sqrt{3})^2 + BH^2$

$\Rightarrow BH^2 = 63$

$BH = 3\sqrt{3}$

مثلث قائم‌الزاویه

$\Rightarrow \hat{B} = \frac{180-90}{2} = 45^\circ$

$\hat{B} = 45^\circ$

۳

الف) $\tan 60^\circ = \frac{50\sqrt{3}}{BC}$

در مثلث ABC ~~$BC = 50\sqrt{3}\times\frac{\sqrt{3}}{\sqrt{2}}$~~

~~$BC = 50\times 3$~~

$BC = 150$ (I)

در مثلث ABD:

$\tan 45^\circ = \frac{50\sqrt{3}}{BD} \Rightarrow BD = \frac{50\sqrt{3}}{\sqrt{3}}$

$BD = 50$ (II)

از رابطه I و II استفاده می‌کنیم

$BC - BD = DC \Rightarrow 150 - 50 = 100$

$DC = 100$

ب) در مثلث ABC: $\tan 60^\circ = \frac{BC}{AB}$

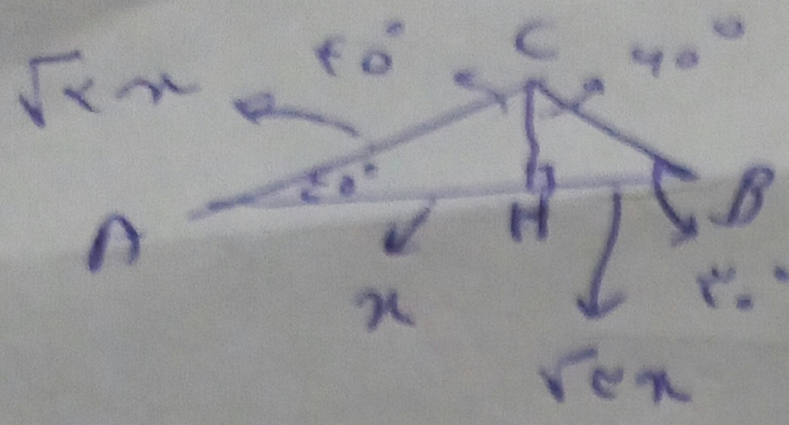
$\sqrt{3} = \frac{BC}{2}$ $BC = 2\sqrt{3}$

در مثلث DBC داریم:

$DC^2 = PB^2 + BC^2$ $DC^2 = 1 + 12 = 13$

$DC = \sqrt{13}$ $\rightarrow \sin\alpha = \frac{BD}{DC} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

(۵) ارتفاع برج AB را بدست آورید:



$CH = AH = x$ — در مثلث ACH

$AC = \sqrt{2}x$

در مثلث HCB

$$\tan 60 = \frac{HB}{HC} \Rightarrow \sqrt{3} = \frac{HB}{x} \quad HB = \sqrt{3}x$$

$$AB = AH + HB = x + \sqrt{3}x = x(1+\sqrt{3})$$

$$\frac{AB}{AC} = \frac{x(1+\sqrt{3})}{x\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{2}$$

---

(۶) ب) $S_{ABE} = \frac{1}{2} \times 2 \times 4 \times \sin\alpha_1$

$S_{BCD} = \frac{1}{2} \times 4 \times 3 \times \sin\alpha_2$

$\alpha_1 = \alpha_2$ متقابل به رأس

$$\frac{S_{ABE}}{S_{BCD}} = \frac{4}{9}$$

نسبت دو مساحت $\frac{4}{9}$

مساحت مثلث‌ها ← بر اساس فرمول‌های

$$\begin{cases} S_{ABE} = \frac{1}{2}\sin\alpha_1 \times BE \times AB \\ S_{BCD} = \frac{1}{2}\sin\alpha_2 \times BC \times BD \end{cases}$$

---

(۶) الف) مساحت متوازی‌الاضلاع $= a$ ضلع $\times b$ ضلع $\times \sin\alpha$

$$S = 3 \times 4 \times \sin 120^\circ = 3 \times 4 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$$

$\sin 120^\circ \to \frac{\sqrt{3}}{2}$

ب) $S_{\text{متوازی‌الاضلاع}} = \frac{1}{2} \times$ قطر۱ $\times$ قطر۲ $\times \sin\alpha$

$$S = \frac{1}{2} \times 4 \times 3 \times \sin 120^\circ = 12\sqrt{3}$$

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(۷)

$$\tan\theta = \frac{\Delta Y}{\Delta x} = \frac{4-2}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$$

$$1 + \tan^2\theta = \frac{1}{\cos^2\theta} \qquad 1 + \frac{1}{4} = \frac{1}{\cos^2\theta} \Rightarrow \cos^2\theta = \pm\frac{2\sqrt{5}}{5}$$

$\cos\theta = \frac{2\sqrt{5}}{5}$ ← تانژانت مثبت است نه فقط + کسینوس

(۸)

$y = mx + b$ $\quad m = \tan\theta = \tan 120^\circ = -\sqrt{3}$

$2y = 2x + 5 \xrightarrow{x=0} 2y = 5 \Rightarrow y = 2.5$

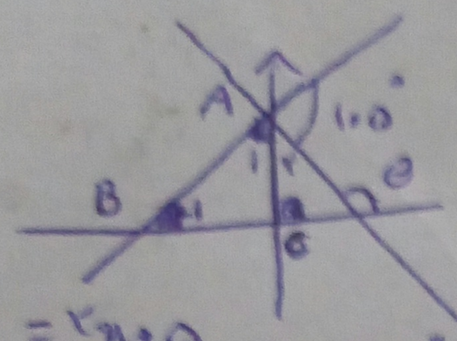
$y = 0 \Rightarrow 2x = 5 \Rightarrow x = -2.5$

ΔAOB متساوی‌الساقین $\Rightarrow \hat{A}_1 = \hat{B}_1 = 45^\circ$

$\Rightarrow \hat{A}_2 = 180 - (45 + 105) = 30^\circ$

$\hat{A}_2 = 30^\circ$

$\Rightarrow \hat{\theta} = 30^\circ + 90^\circ = 120^\circ$ زاویه خارجی



$y_2 = \frac{5}{2}x + \frac{5}{2}$

$y_1 = y_2$ $\quad x + \frac{5}{2} = mx + b \xrightarrow{m = -\sqrt{3}}$

$x + \frac{5}{2} = -\sqrt{3}x + b \xrightarrow{x=0}$

$b = -\frac{5}{2}$

$mb = \frac{-5}{4} \times -\sqrt{3}$

$mb = \frac{5\sqrt{3}}{4}$

(۹)

$F(x) = -\cos x + \cos x - (-\cot x)$

$F(x) = \cot x \Rightarrow F\left(\frac{5}{4}\pi\right) = \cot\left(\frac{5}{4}\pi\right) = -\sqrt{3}$

(۱۰)

$$\frac{\sin\left(\frac{\pi}{2} + x\right) + \cos\left(\frac{3\pi}{2} - x\right)}{\cos(\pi + x) - \sin(-x)} = \frac{-\cos x + \sin x}{-\cos x + \sin x} = 1$$

$\sin\left(\frac{\pi}{2} + x\right) \to -\cos x$

$\cos\left(\frac{3\pi}{2} - x\right) \to \sin(x)$

$\cos(\pi + x) \to -\cos(x)$

$-\sin(-x) \to -\sin(x)$