پارسا ضیائی

19,5

شماره تکلیف ۲۰ - دهم پسر A

Subject: Date

۱- الف) $(\sin 135^\circ + \sin 300^\circ)(\cos 315^\circ - \cos 150^\circ) = \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\right)$

$\sin 135^\circ = \frac{\sqrt{2}}{2}$, $\sin 300^\circ = -\frac{\sqrt{3}}{2}$, $\cos 315^\circ = \frac{\sqrt{2}}{2}$, $\cos 150^\circ = -\frac{\sqrt{3}}{2}$

$= \frac{2}{4} - \frac{3}{4} = -\frac{1}{4}$ ✓ (۲)

ب) $\frac{1 + 3\tan^2 210^\circ}{4\cos^2 330^\circ + 2\cos^2 225^\circ}$

$\tan 210^\circ = \frac{1}{\sqrt{3}}$, $4\cos^2 330^\circ = 4 \times \frac{3}{4}$, $2\cos^2 225^\circ = 2 \times \frac{1}{2}$

$\rightarrow \frac{1 + 3 \times \frac{1}{3}}{3 + 1} = \frac{1}{2}$ ✓

۲- الف) $\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{2\sqrt{3}}{\frac{\sqrt{2}}{2}} = \frac{\sqrt{6}}{\sin C} \Rightarrow \sin \hat{C} = \frac{1}{2} \Rightarrow \hat{C} = \begin{cases} 30^\circ \checkmark \\ 150^\circ \times \end{cases}$ (۲)

$\frac{HC}{AC} = \cos 30^\circ \Rightarrow AC \times \cos 30^\circ = HC \Rightarrow HC = 2\sqrt{3} \times \frac{\sqrt{3}}{2} = 3$ ✓

ب) $HC = AC \times \cos 30^\circ \Rightarrow 9 = AC \times \frac{\sqrt{3}}{2} \Rightarrow AC = 6\sqrt{3}$

$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{6\sqrt{3}}{\sin B} = \frac{3\sqrt{6}}{\frac{1}{2}} \Rightarrow \sin \hat{B} = \frac{\sqrt{2}}{2} \Rightarrow \hat{B} = \begin{cases} 45^\circ \checkmark \\ 135^\circ \times \end{cases}$ ✓

۳- الف) $\frac{CB}{AB} = \cot \hat{C} \Rightarrow \frac{CB}{50\sqrt{3}} = \sqrt{3} \Rightarrow CB = 150$

$\frac{BD}{AB} = \cot \hat{D} \Rightarrow \frac{BD}{50\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow BD = 50$

$x = BC - BD = 150 - 50 = 100$ ✓ (۲)

ب) $\frac{BC}{AB} = \frac{\tan 60^\circ}{\tan \hat{A}} \Rightarrow \frac{BC}{2} = \sqrt{3} \Rightarrow BC = 2\sqrt{3}$

$CD^2 = DB^2 + BC^2 \Rightarrow CD^2 = 1 + 12 \Rightarrow CD = \begin{cases} +\sqrt{13} \checkmark \\ -\sqrt{13} \times \end{cases}$

$\sin \alpha = \frac{BD}{CD} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$ ✓

۴- $\hat{B} = 180 - (45 + 105) = 30$

$\sin 105^\circ = \sin(60^\circ + 45^\circ) = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

$\frac{AB}{\sin C} = \frac{AC}{\sin B} \Rightarrow \frac{AB}{AC} = \frac{\sin C}{\sin B} = \frac{\frac{\sqrt{6}+\sqrt{2}}{4}}{\frac{1}{2}} = \frac{\sqrt{6}+\sqrt{2}}{2}$ ✓ (۲)

۵- $\frac{S_{ABE}}{S_{BCD}} = \frac{\frac{1}{2} \times 2 \times 4 \times \sin \hat{B}_1}{\frac{1}{2} \times 3 \times 4 \times \sin \hat{B}_2}$ $\hat{B}_1 = \hat{B}_2 \rightarrow \frac{\frac{1}{2} \times 2 \times 4}{\frac{1}{2} \times 3 \times 4} = \frac{4}{6}$ ✓ (۲)

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۶- (۲)

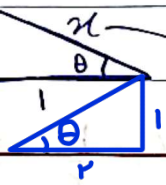
الف) $۴ \times ۳^{۲} \times \sin ۱۲۰^\circ = ۴ \times ۳ \times \frac{\sqrt{۳}}{۲} = ۶\sqrt{۳}$ ✓

ب) $\frac{۱}{۳} \times ۳^{۲} \times ۴^{۲} \times \sin ۱۲۰^\circ = ۶ \times \frac{\sqrt{۳}}{۲} = ۳\sqrt{۳}$ ✓

۷- (۱/۵)

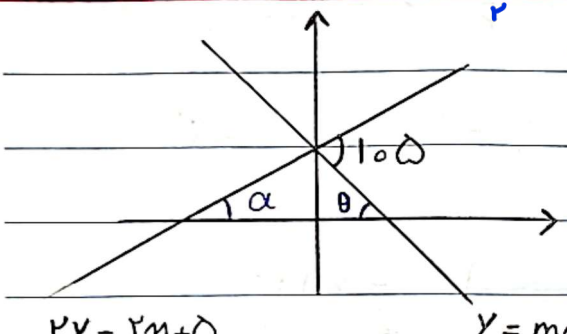
شیب خط $= \frac{\Delta y}{\Delta x} = \frac{۴-۲}{۳-(-۱)} = \frac{۲}{۴} = \frac{۱}{۲}$

$\tan\theta = \frac{۱}{۲} \Rightarrow$ ضلع مقابل به مجاور



$\Rightarrow \cos\theta = \begin{cases} \frac{۱}{\sqrt{۵}} \checkmark \\ -\frac{۱}{\sqrt{۵}} \times \end{cases}$ $\boxed{\frac{۲}{\sqrt{۵}}}$

۸- (۲)



$۲y = ۲x + ۵ \Rightarrow y = x + ۲/۵$

$\tan\alpha = ۱ \Rightarrow \alpha = \begin{cases} ۴۵^\circ \checkmark \\ ۲۲۵^\circ \times \end{cases}$

$\alpha = ۴۵ \Rightarrow \theta = ۶۰^\circ$

$y = mx + b \rightarrow m = \tan(۱۱۰ - \theta) = \tan ۱۲۰ = -\sqrt{۳}$

چون هر دو خط در یک نقطه محور y را قطع کردند،

پس عرض از مبدا برابر خواهند در نتیجه $b = ۲/۵$

$mb = -\sqrt{۳} \times \frac{۵}{۲} = \frac{-۵\sqrt{۳}}{۲}$ ✓

۹- (۲)

$f(x) = \cos(\pi + x) + \sin(\frac{\pi}{۲} - x) - \tan(\frac{۳\pi}{۲} + x) \Rightarrow$

$f(x) = -\cos x + \cos x + \cot x = \cot x$

$f(\frac{۵\pi}{۶}) = \cot\frac{۵\pi}{۶} = -\sqrt{۳}$ ✓

۱۰- (۲)

$$\frac{\sin(\frac{۱۷\pi}{۲} + x) + \cos(\frac{۱۵\pi}{۲} - x)}{\cos(۳\pi + x) - \sin(۶\pi - x)} = \frac{\sin(\frac{\pi}{۲} + x) + \cos(\frac{۳\pi}{۲} - x)}{\cos(\pi + x) - \sin(۲\pi - x)}$$

$$= \frac{\cos x + (-\sin x)}{-\cos x + \sin x} = \frac{\cos x - \sin x}{\sin x - \cos x} = -۱$$ ✓