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$$\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = \left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} - \frac{2}{4} = -\frac{1}{2} \checkmark$$

$$\frac{10 \left(\frac{\sqrt{2}}{2}\right)^2}{4 \left(\frac{\sqrt{2}}{2}\right)^2 + 4 \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{2}{2+2} = \frac{1}{2} \checkmark$$

$$DH = \sqrt{2} \sin \alpha \Rightarrow \sqrt{3} \Rightarrow (DH)^2 + (HC)^2 = (DC)^2$$

$$\frac{\sqrt{2}}{2} \Rightarrow 3 + (HC)^2 = 12 \Rightarrow HC = \sqrt{3} \boxed{3}$$

$$AC \cos 30 = 9 \Rightarrow DC = 4\sqrt{3} \xrightarrow{\text{فيثاغورس}} (DH)^2 + (10-11)^2 = DH^2 \Rightarrow DH = 2\sqrt{3}$$

$$\Rightarrow \sin B = \frac{2\sqrt{3}}{4\sqrt{3}} = \frac{\sqrt{2}}{2} \Rightarrow \boxed{B = 45^\circ} \checkmark$$

$$DC \sin 30 = 10\sqrt{3} \Rightarrow DC = 100\sqrt{3} \Rightarrow BC = DC \cos 30 = 100$$

$$BD \sin 40 = 10\sqrt{3} \Rightarrow BD = 100 \Rightarrow BP = DC \cos 40 = 100 \Rightarrow PC = 100 - 100 = 0 \checkmark$$

$$\hat{C} = 40^\circ \Rightarrow DC \sin 40 = 2 \Rightarrow DC = 4 \Rightarrow BC = 2\sqrt{3}$$

$$\xrightarrow{\text{فيثاغورس}} (PC)^2 + (PB)^2 = (BC)^2 = 1 + 12 \Rightarrow PC = \sqrt{13} \Rightarrow \sin \alpha = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13} \checkmark$$

$$\frac{AB}{\sin C} = \frac{BC}{\sin B} \Rightarrow \frac{AB}{BC} = \frac{\sin C}{\sin B} = \frac{\sin 100}{\sin 30} = \frac{\sin 100 \cos 30 - \cos 100 \sin 30}{\sin 30}$$

$$= \frac{\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}}{\frac{1}{2}} = \frac{\frac{\sqrt{6} + \sqrt{2}}{2}}{\frac{1}{2}} = \sqrt{6} + \sqrt{2} \checkmark$$

$$\frac{S_{ABE}}{S_{BCD}} = \frac{\frac{1}{2} \times 2 \times 4 \times \sin B}{\frac{1}{2} \times 4 \times 4 \times \sin B} = \frac{1}{12} \Rightarrow \boxed{\frac{4}{9}} \checkmark$$

$$S_{\text{مربع (ABCD)}} = 4 (S_{\triangle ABC}) = 4 \times 3 \times \sin 120 = 4 \times 3 \times \frac{\sqrt{3}}{2} = 6\sqrt{3} \checkmark$$

$$S_{\text{مربع (ABCD)}} = 4 \times 4 \times \sin 120 \times \frac{1}{2} = 4\sqrt{3} \checkmark$$

$$m \tan \alpha = \frac{r-1}{r-(-1)} = \frac{1}{2}$$

(2) ✓

$$\tan^2 \alpha + 1 = \frac{1}{\cos^2 \alpha} \Rightarrow \frac{1}{4} + 1 = \frac{1}{\cos^2 \alpha} \Rightarrow \cos^2 \alpha = \frac{4}{5} \Rightarrow \cos \alpha = \frac{2\sqrt{5}}{5}$$

$$y = y_0 + d \Rightarrow y = x + r_1 d \Rightarrow m = 1 \Rightarrow \tan \alpha = 1 \Rightarrow \alpha = 45^\circ$$

(1,5) ✓

$$m_1 = -\tan(100^\circ - 45^\circ) = \tan 40^\circ = \frac{\sqrt{3}}{3}$$

$$b_1 = b_2 \rightarrow \text{خطوط موازية} \Rightarrow b_1 = r_1 d$$

$$m b = \frac{d}{r} \alpha = \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$$\boxed{-\frac{5\sqrt{3}}{2}}$$

$$f(x) = -\cos x + \cos x + \cot x \Rightarrow f(x) = \cot x$$

(2) ✓

$$\Rightarrow x = \frac{2\pi}{4} \Rightarrow f(x) = \cot \frac{2\pi}{4} = \cot 180^\circ = -\sqrt{3}$$

1- ابتدا جواب را در کتب مرجع و در اینترنت به شکل زیر یافت می شود:

$$\frac{\sin(\pi + \frac{\pi}{4} + x) + \cos(\pi + \frac{\pi}{4} - x)}{\cos(\pi + \pi + x) - \sin(\pi + \pi - x)} = \frac{\sin(\frac{\pi}{4} + x) + \cos(\frac{\pi}{4} - x)}{\cos(\pi + x) - \sin(\pi - x)}$$

$$= \frac{\cos x - \sin x}{-(\cos x + \sin x)} = \frac{\cos x - \sin x}{-1(\cos x + \sin x)} = -1$$