

$$(1! + 3! + 5! + \dots + 25!) (2! + 4! + 6! + \dots + 24!) = (1 + 4 + 12 + 50 + 40 + \dots + \dots)$$

$$(2 + 24 + 720 + 6420 + \dots + \dots) \xrightarrow{\text{تقریباً}} (2 + 4)(1 + 4) = 4 \times 5 = 20 \xrightarrow{\text{تقریباً}} 20 \checkmark$$

الف) $(4 + \sqrt{5})^{-\frac{1}{2}} \times (1 + \sqrt{5})^{\frac{1}{2}} \Rightarrow (1 + \sqrt{5})^2 = 1 + 2\sqrt{5} + 5 = 6 + 2\sqrt{5} = 2(3 + \sqrt{5})$
 $4 + \sqrt{5} = \frac{(1 + \sqrt{5})^2}{2} \rightarrow (4 + \sqrt{5})^{-\frac{1}{2}} \Rightarrow \left(\frac{(1 + \sqrt{5})^2}{2}\right)^{-\frac{1}{2}} \Rightarrow 2^{\frac{1}{2}} \times (1 + \sqrt{5})^{-\frac{1}{2}}$
 $\Rightarrow (1 + \sqrt{5})^{-\frac{1}{2}} \times (1 + \sqrt{5})^{\frac{1}{2}} \times 2^{\frac{1}{2}} = \sqrt{2} \checkmark$
 ب) $(\sqrt{3} - \sqrt{5} - \sqrt{3 + \sqrt{5}})^2 =$

$$(3 - 5) + (3 + 5) - 2\sqrt{(3 - 5)(3 + 5)} = 4 - 2\sqrt{4} = 4 - 4 = 0 \xrightarrow{\text{عبارت صفری}} \sqrt{3 - \sqrt{5}} - \sqrt{3 + \sqrt{5}} = -\sqrt{2}$$

$$\rightarrow (-\sqrt{2}) \left(\frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} \right) = -\frac{\sqrt{2}(\sqrt{5} + \sqrt{3})}{\sqrt{5} + \sqrt{3}} = -1 \checkmark$$

الف) $\frac{\sqrt{8} + \sqrt{2}}{5 - \sqrt{4}} - 2 \left(\sqrt[4]{9} - 1 \right)^{-1} \times \frac{2\sqrt{2} + 2\sqrt{3}}{5 - \sqrt{4}} \times \frac{5 + \sqrt{4}}{5 + \sqrt{4}} = \frac{10\sqrt{2} + 10\sqrt{3} + 2\sqrt{12} + 2\sqrt{18}}{25 - 4}$
 $= \frac{19\sqrt{2} + 19\sqrt{3}}{19} = \sqrt{2} + \sqrt{3}$
 $\frac{1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{\sqrt{3} + 1}{2} \Rightarrow \sqrt{2} + \sqrt{3} - \frac{\sqrt{3} + 1}{2} = \sqrt{2} + \sqrt{3} - \frac{\sqrt{3} - 1}{2}$

ب) $\frac{\sqrt{2} - 1}{4 + \sqrt{3}} + (2 - \sqrt{3})^{-1} \rightarrow \frac{2\sqrt{3} - 1}{4 + \sqrt{3}} \times \frac{4 - \sqrt{3}}{4 - \sqrt{3}} = \frac{12\sqrt{3} - 4 - 9 + \sqrt{3}}{14 - 3} = \frac{13\sqrt{3} - 13}{13} = \sqrt{3} - 1 \checkmark$
 $\frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3} \Rightarrow 2 + \sqrt{3} + \sqrt{3} - 1 = 2\sqrt{3} + 1 \checkmark = \sqrt{3} - 1$

الف) $\frac{\sqrt{1 + \sqrt{3}} + \sqrt{\sqrt{3} - 1}}{\sqrt{\sqrt{3} - \sqrt{2}}} - 2 = 4 + 2\sqrt{4} - 2 = 4 + 2\sqrt{4} = 2(2 + \sqrt{4})$
 $\frac{\sqrt{1 + \sqrt{3}} + \sqrt{\sqrt{3} - 1}}{\sqrt{\sqrt{3} - \sqrt{2}}} \times \frac{\sqrt{\sqrt{3} + \sqrt{2}}}{\sqrt{\sqrt{3} + \sqrt{2}}} = \frac{4 + 2\sqrt{4}}{1}$

ب) $\sqrt[4]{3\sqrt{2}} \times \sqrt[4]{142} \times \sqrt[4]{4\sqrt{2}} = \sqrt[4]{3^4 \times 2^4 \times 142^4 \times 4^4 \times 2^4} = \sqrt[4]{3^4 \times 2^4 \times 142^4 \times 2^4 \times 2^4} = \sqrt[4]{3^4 \times 2^8 \times 142^4} = 2 \times 2 \times 142 = 284 \checkmark$

$\frac{3^M (1 + 3 + 3^2 + 3^3 + 3^4 + 3^5)}{2^{M-2} (1 + 2 + 2^2 + 2^3 + 2^4 + 2^5)} = \frac{3^M \times \frac{3^6 - 1}{3 - 1}}{2^{M-2} (2^6 - 1)} = \frac{3^M \times 343}{2^{M-2} \times 63}$
 $\frac{343}{63} = \frac{52}{9} \times \frac{3^M}{2^{M-2}} = \frac{52}{9} \times 4 \times \left(\frac{3}{2}\right)^M = \frac{208}{9} \left(\frac{3}{2}\right)^M = 52$
 $\Rightarrow \left(\frac{3}{2}\right)^M = 52 \times \frac{9}{208} = \frac{9}{4} \Rightarrow \left(\frac{3}{2}\right)^M = \frac{9}{4} \Rightarrow M = 2 \checkmark$

$$\sqrt[5]{\sqrt{9}-r} = a \quad b = \sqrt[5]{\sqrt{4}+r}$$

$$(a-b)^5 (a+b)^5 = + (r-\sqrt{3}) \Rightarrow (a^5-b^5)^5 \Rightarrow ((\sqrt{\sqrt{9}-r}-\sqrt{\sqrt{4}+r}))^5)^5$$

$$(r\sqrt{4}-r\sqrt{r})^5 = + (r-\sqrt{3})$$

$$r^5+1 - \sqrt{1} \sqrt{r} = + (r-\sqrt{3}) \Rightarrow r^5-14\sqrt{r} = + (r-\sqrt{3}) \Rightarrow t=14 \checkmark$$

$$a + \frac{1}{a} = n \Rightarrow (n+\sqrt{r})^r (n-\sqrt{r})^r = (n^2-r)^r \Rightarrow n = a + \frac{1}{a} = \sqrt{r-\sqrt{r}} + \sqrt{r+\sqrt{r}}$$

$$\sqrt{r-\sqrt{r}} = a, \quad \sqrt{r+\sqrt{r}} = \frac{1}{a}$$

$$n^2 = (r\sqrt{r}) + (r+\sqrt{r}) + r \sqrt{(r-\sqrt{r})(r+\sqrt{r})} = 8+r = 9$$

$$\Rightarrow (n^2-r)^r = (9-r)^r = 14 \Rightarrow t=8 \checkmark$$

$$A = \sqrt[5]{f^3 \sqrt{14}} \left(\frac{1}{r}\right)^{-\frac{f}{r}} = \sqrt[5]{f^3 \times 14} \times r^{\frac{f}{r}} = \sqrt[5]{f^3 \times r^f} \times r^{\frac{f}{r}} = \sqrt[5]{r^{10}} \times r^{\frac{f}{r}} = r^2 \times r^{\frac{f}{r}} = r^{\frac{r+f}{r}} = r^{\frac{4}{r}} = r^2 = f \checkmark$$

$$(r \cancel{f})^{-\frac{1}{r}} \xrightarrow{A=f} (r \times f)^{-\frac{1}{r}} = 1^{-\frac{1}{r}} = \frac{1}{1^{\frac{1}{r}}} = \frac{1}{\sqrt[r]{1}} = \frac{1}{r} \checkmark$$

$$a^{\frac{1}{v}} = r^u \times a^{\frac{10}{v}} \Rightarrow \frac{a^{\frac{1}{v}}}{a^{\frac{10}{v}}} = r^u \Rightarrow a^{-\frac{9}{v}} = r^u \Rightarrow a^{-r} = r^u \Rightarrow \frac{1}{a^r} = r^u$$

$$\Rightarrow \frac{1}{a} = r\sqrt{r}$$

$$\frac{\frac{1}{a}-r}{1+\sqrt{r}} \xrightarrow{\frac{1}{a}=r\sqrt{r}} \frac{r\sqrt{r}-r}{\sqrt{r}+1} \times \frac{\sqrt{r}-1}{\sqrt{r}-1} = \frac{r(\sqrt{r}-1)^2}{r-1} = \frac{r(1+r-r\sqrt{r})}{r-1} = \frac{r-4\sqrt{r}}{r} = 4-2\sqrt{r} \checkmark$$

$$A = \sqrt{n+a} \quad A-B=r$$

$$B = \sqrt{n-f} \quad A^2-B^2 = (n+a) - (n-f) = f+a \quad (A-A)(B-B) = f+a$$

$$r(A+B) = f+a \Rightarrow A+B = \frac{f+a}{r} \Rightarrow A+B-r = \frac{f+a}{r} - r = \frac{a}{r} \checkmark$$

$$\left(\frac{\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} \right)^2 - 2 = \frac{\sqrt{2}(\sqrt{\sqrt{3}+\sqrt{2}})}{\sqrt{\sqrt{3}-\sqrt{2}}} - 2 = \sqrt{2} \left(\sqrt{\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}} \right) - 2 =$$

$$\rightarrow A^2 = 2\sqrt{3} + 2\sqrt{2} \rightarrow A = \sqrt{2}(\sqrt{\sqrt{3}+\sqrt{2}})$$

$$\sqrt{2} \sqrt{(\sqrt{\sqrt{3}+\sqrt{2}})^2} - 2 = \sqrt{2} + 2 - 2 = \boxed{\sqrt{2}}$$

لغالباً مردن!