نام و نام خانوادگی ...محمدحسین حسینی‌بیگی... پاسخنامه تشریحی تکلیف شماره ...۲۱... کلاس ...دهم ب...

۱

$$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+26!) = (1+6+120+5040+\dots+\dots0)$$

$$(2+24+720+40320+\dots+\dots0) \xrightarrow{\text{رقم یکان}} (2+4)(1+6) = 6\times7 = 42 \xrightarrow{\text{رقم یکان}} \boxed{2}$$

۲

الف) $(4+\sqrt7)^{-\frac14}\times(1+\sqrt7)^{\frac12} \Rightarrow (1+\sqrt7)^2 = 1+2\sqrt7+7 = 8+2\sqrt7 = 2(4+\sqrt7)$

$4+\sqrt7 = \frac{(1+\sqrt7)^2}{2} \longrightarrow (4+\sqrt7)^{-\frac14} \Rightarrow \left(\frac{(1+\sqrt7)^2}{2}\right)^{-\frac14} \Rightarrow 2^{\frac14}\times(1+\sqrt7)^{-\frac12}$

$\Rightarrow (1+\sqrt7)^{-\frac12}\times(1+\sqrt7)^{\frac12}\times2^{\frac14} = \boxed{\sqrt[4]{2}}$

ب) $\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)^2 =$

$(3-\sqrt5)+(3+\sqrt5)-2\sqrt{(3-\sqrt5)(3+\sqrt5)} = 6-2\sqrt4 = 6-4 = 2 \xrightarrow{\text{عبارت منفی}} \sqrt{3-\sqrt5}-\sqrt{3+\sqrt5} = -\sqrt2$

$\longrightarrow (-\sqrt2)\left(\frac{\sqrt5+\sqrt2}{2+\sqrt{10}}\right) = -\frac{2+\sqrt{10}}{\sqrt{10}+2} = \boxed{-1}$

۳

الف) $\frac{\sqrt8+\sqrt{27}}{5-\sqrt6} - 2\left(\sqrt[4]{9}-1\right)^{-1}$

$\frac{2\sqrt2+3\sqrt3}{5-\sqrt6}\times\frac{5+\sqrt6}{5+\sqrt6} = \frac{10\sqrt2+15\sqrt3+2\sqrt{12}+3\sqrt{18}}{25-6}$

$= \frac{19\sqrt2+19\sqrt3}{19} = \sqrt2+\sqrt3$

$\frac{1}{\sqrt3-1}\times\frac{\sqrt3+1}{\sqrt3+1} = \frac{\sqrt3+1}{2}$ ✓ $\Rightarrow \sqrt2+\sqrt3-\cancel{2}\left(\frac{\sqrt3+1}{\cancel{2}}\right) = \sqrt2+\sqrt3-\sqrt3-1 = \boxed{\sqrt2-1}$

ب) $\frac{\sqrt{27}-1}{4+\sqrt3} + (2-\sqrt3)^{-1}$

$\frac{3\sqrt3-1}{4+\sqrt3}\times\frac{4-\sqrt3}{4-\sqrt3} = \frac{12\sqrt3-4-9+\sqrt3}{16-3} = \frac{13\sqrt3-13}{13} = \sqrt3-1$

$\frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3} = \frac{2+\sqrt3}{4-3} = 2+\sqrt3$ ✓ $\Rightarrow 2+\sqrt3+\sqrt3-1 = \boxed{2\sqrt3+1}$

۴

الف) $\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}} - 2 = 6+2\sqrt6-2 = 4+2\sqrt6 = \boxed{2(2+\sqrt6)}$

$\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}\times\frac{\sqrt{\sqrt3+\sqrt2}}{\sqrt{\sqrt3+\sqrt2}} = \frac{6+2\sqrt6}{1}$

ب) $\sqrt[4]{\sqrt[3]{2^8}}\times\sqrt[4]{162}\times\sqrt{4\sqrt[6]{2}} = \sqrt[12]{2^8}\times\sqrt[4]{2\times3^4}\times\sqrt[12]{2\times2^{12}} = \sqrt[12]{2^8\times2\times2^3}\times3\times2 = 2\times3\times2 = \boxed{12}$

۵

$$\frac{3^n(1+3+3^2+3^3+3^4+3^5)}{2^{n-2}(1+2+2^2+2^3+2^4+2^5)} = \frac{3^n\times\frac{3^6-1}{3-1}}{2^{n-2}(2^6-1)} = \frac{3^n\times364}{2^{n-2}\times63}$$

$$\frac{364}{63} = \frac{52}{9}\times\frac{3^n}{2^{n-2}} = \frac{52}{9}\times4\times\left(\frac32\right)^n = \frac{208}{9}\left(\frac32\right)^n = 52$$

$$\Rightarrow \left(\frac32\right)^n = 52\times\frac{9}{208} = \frac94 \Rightarrow \left(\frac32\right)^n = \frac94 \Rightarrow \boxed{n=2}$$

$\sqrt[۴]{\sqrt{۹}-۲} = a \qquad b = \sqrt[۴]{\sqrt{۹}+۲}$

$(a-b)^۴(a+b)^۴ = t(۲-\sqrt{۳}) \Rightarrow (a^۲-b^۲)^۴ \Rightarrow \left(\left(\sqrt{\sqrt{۹}-۲}-\sqrt{\sqrt{۹}+۲}\right)^۲\right)^۲$ ۶

$(۲\sqrt{۹}-۲\sqrt{۲})^۲ = t(۲-\sqrt{۳})$

$۲۴+۸-۸\sqrt{۱۲} = t(۲-\sqrt{۳}) \Rightarrow ۳۲-۱۶\sqrt{۳} = t(۲-\sqrt{۳}) \Rightarrow \boxed{t=۱۶}$

$a+\frac{۱}{a} = x \Rightarrow (x+\sqrt{۲})^۲(x-\sqrt{۲})^۲ = (x^۲-۲)^۲ \Big\} \Rightarrow x = a+\frac{۱}{a} = \sqrt{۲-\sqrt{۳}}+\sqrt{۲+\sqrt{۳}}$

$\sqrt{۲-\sqrt{۳}} = a \ , \ \sqrt{\sqrt{۳}+۲} = \frac{۱}{a} \longleftarrow$ ۷

$x^۲ = (۲-\sqrt{۳})+(۲+\sqrt{۳})+۲\sqrt{(۲-\sqrt{۳})(۲+\sqrt{۳})} = ۴+۲ = ۶$

$\Rightarrow (x^۲-۲)^۲ = (۶-۲)^۲ = ۱۶ \Rightarrow \boxed{t=۴}$

$A = \sqrt[۵]{۴\sqrt[۳]{۱۶}}\left(\frac{۱}{۲}\right)^{-\frac{۴}{۳}} = \sqrt[۱۵]{۴^۳\times ۱۶}\times ۲^{\frac{۴}{۳}} = \sqrt[۱۵]{۲^۶\times ۲^۴}\times ۲^{\frac{۴}{۳}} = \sqrt[۱۵]{۲^{۱۰}}\times ۲^{\frac{۴}{۳}}$

$= \sqrt[۳]{۲^۲}\times ۲^{\frac{۴}{۳}} = ۲^{\frac{۲}{۳}}\times ۲^{\frac{۴}{۳}} = ۲^{\frac{۶}{۳}} = ۲^۲ = ۴$ ۸

$(۲A)^{-\frac{۱}{۳}} \xrightarrow{A=۴} (۲\times ۴)^{-\frac{۱}{۳}} = ۸^{-\frac{۱}{۳}} = \frac{۱}{۸^{\frac{۱}{۳}}} = \frac{۱}{\sqrt[۳]{۸}} = \boxed{\frac{۱}{۲}}$

$a^{\frac{۱}{۷}} = ۲۷\times a^{\frac{۱۵}{۷}} \Rightarrow \frac{a^{\frac{۱}{۷}}}{a^{\frac{۱۵}{۷}}} = ۳^۳ \Rightarrow a^{-\frac{۱۴}{۷}} = ۳^۳ \Rightarrow a^{-۲} = ۳^۳ \Rightarrow \frac{۱}{a^۲} = ۳^۳$

$\Rightarrow \frac{۱}{a} = ۳\sqrt{۳}$ ۹

$\frac{\frac{۱}{a}-۳}{۱+\sqrt{۳}} \xrightarrow{\frac{۱}{a}=۳\sqrt{۳}} \frac{۳\sqrt{۳}-۳}{\sqrt{۳}+۱}\times\frac{\sqrt{۳}-۱}{\sqrt{۳}-۱} = \frac{۳(\sqrt{۳}-۱)^۲}{۳-۱} = \frac{۳(۱+۳-۲\sqrt{۳})}{۲} = \frac{۱۲-۶\sqrt{۳}}{۲} = \boxed{۶-۳\sqrt{۳}}$

$A = \sqrt{x+a} \qquad A-B = ۲$

$B = \sqrt{x-۴} \qquad A^۲-B^۲ = (x+a)-(x-۴) = ۴+a \qquad (A-A)(B-B) = ۴+a$ ۱۰

$۲(A+B) = ۴+a \Rightarrow A+B = \frac{۴+a}{۲} \Rightarrow A+B-۲ = \frac{۴+a}{۲}-۲ = \boxed{\frac{a}{۲}}$