۱)

$(۱! + ۳! + ۵! + \dots + ۲۵!) \xrightarrow{\text{یکان}} ۱ + ۶ = ۷$

$۱! \xrightarrow{\text{یکان}} ۱$

$۳! \xrightarrow{\text{یکان}} ۶$

$۲! \xrightarrow{\text{یکان}} ۲$

$۴! \xrightarrow{\text{یکان}} ۴$

از $۵!$ به بعد چون عامل ۱۰ وجود دارد یکان صفر می‌شود

$(۲! + ۴! + ۶! + \dots + ۲۶!) \xrightarrow{\text{یکان}} ۴ + ۲ = ۶$

یکان حاصلضرب $= ۲$

۲)

الف) $\sqrt[4]{(4+\sqrt{7})^{-1}}\,\sqrt{1+\sqrt{7}} = \sqrt[4]{(4+\sqrt{7})^{-1}}\,\sqrt[4]{(1+\sqrt{7})^{2}}$

$= \sqrt[4]{\dfrac{8+2\sqrt{7}}{4+\sqrt{7}}} = \sqrt[4]{\dfrac{2(4+\sqrt{7})}{\cancel{4+\sqrt{7}}}} = \boxed{\sqrt[4]{2}}$

ب) $\left(\dfrac{\sqrt{2}+\sqrt{5}}{\sqrt{2}(\sqrt{5}+\sqrt{2})}\right)\left(\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}\right)$

$= \left(\dfrac{\sqrt{2}}{2}\right)\left(\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}\right) = \dfrac{\sqrt{6-2\sqrt{5}}-\sqrt{6+2\sqrt{5}}}{2} = \dfrac{\sqrt{(\sqrt{5}-1)^2}-\sqrt{(\sqrt{5}+1)^2}}{2}$

$= \dfrac{\sqrt{5}-1-\sqrt{5}-1}{2} = \dfrac{-2}{2} = \boxed{-1}$

۳)

الف) $\dfrac{\sqrt{8}+\sqrt{27}}{5-\sqrt{6}} - 2(\sqrt[4]{9}-1)^{-1} =$

$\dfrac{(\sqrt{2}+\sqrt{3})(5-\sqrt{6})}{5-\sqrt{6}} - \dfrac{2}{\sqrt{3}-1} = \sqrt{2}+\sqrt{3}-\dfrac{2}{\sqrt{3}-1}$

$\dfrac{2}{\sqrt{3}-1}\times\dfrac{\sqrt{3}+1}{\sqrt{3}+1} = \sqrt{3}+1 \rightarrow \sqrt{2}+\sqrt{3}-\dfrac{2}{\sqrt{3}+1} = \sqrt{2}+\sqrt{3}-\sqrt{3}-1 =$

$\boxed{\sqrt{2}-1}$

ب) $\frac{\sqrt{۲۷}-۱}{۴+\sqrt{۳}} + (۲-\sqrt{۳})^{-۱}$

$\frac{(\sqrt{۳}-۱)(۴+\sqrt{۳})}{۴+\sqrt{۳}} + \frac{۱}{۲-\sqrt{۳}} = \sqrt{۳}-۱+\frac{۱}{۲-\sqrt{۳}}$

$= \frac{(\sqrt{۳}-۱)(۲-\sqrt{۳})+۱}{۲-\sqrt{۳}} \times \frac{۲+\sqrt{۳}}{۲+\sqrt{۳}} = \frac{۲\sqrt{۳}-۱+۲+\sqrt{۳}}{۴-۳} = \boxed{۲\sqrt{۳}+۱}$

۴)

الف) $\frac{\sqrt{۱+\sqrt{۳}}+\sqrt{\sqrt{۳}-۱}}{\sqrt{\sqrt{۳}-\sqrt{۲}}}$

$A = \sqrt{۱+\sqrt{۳}}+\sqrt{\sqrt{۳}-۱} \xrightarrow{\text{توان ۲}} \sqrt{۳}+۱+\sqrt{۳}-۱+۲\sqrt{(\sqrt{۳}+۱)(\sqrt{۳}-۱)}$

$= ۲\sqrt{۳}+۲\sqrt{۲} = ۲(\sqrt{۳}+\sqrt{۲}) = A^۲$

$\Rightarrow A = \sqrt{۲(\sqrt{۳}+\sqrt{۲})}$

$\rightarrow \frac{\sqrt{۲(\sqrt{۳}+\sqrt{۲})}}{\sqrt{\sqrt{۳}-\sqrt{۲}}} = \sqrt{\frac{۲(\sqrt{۳}+\sqrt{۲})}{\sqrt{۳}-\sqrt{۲}}} \times \sqrt{\frac{\sqrt{۳}+\sqrt{۲}}{\sqrt{۳}+\sqrt{۲}}} = \sqrt{\frac{۲(\sqrt{۳}+\sqrt{۲})^۲}{۱}}$

$= \sqrt{۲(۵+۲\sqrt{۶})} = \sqrt{۱۰+۴\sqrt{۶}} = \sqrt{(\sqrt{۶}+۲)^۲} = \sqrt{۶}+۲$

$\rightarrow \frac{\sqrt{۱+\sqrt{۳}}+\sqrt{\sqrt{۳}-۱}}{\sqrt{\sqrt{۳}-\sqrt{۲}}} - ۲ = \sqrt{۶}+\cancel{۲}\,۲-۲ = \sqrt{۶}$

ب) $\sqrt[۴]{\sqrt[۳]{۲^۸}} \times \sqrt[۴]{۱۶۲} \times \sqrt{۴\sqrt[۶]{۲}} =$

$\sqrt[۱۲]{۲^۸} \times \sqrt[۴]{۳^۴\times۲} \times \sqrt[۱۲]{۲^{۱۳}} = \sqrt[۱۲]{۲^۸} \times \sqrt[۱۲]{۳^{۱۲}\times۲^۳} \times \sqrt[۱۲]{۲^{۱۳}}$

$= \sqrt[۱۲]{۳^{۱۲}\times۲^{۲۴}} = ۳\times۲^۲ = \boxed{۱۲}$

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5)

$$\frac{3^x(1+3+3^2+\dots+3^5)}{2^x(2^{-2}+2^{-1}+1+2+2^2+2^3)} = 52$$

$$\Rightarrow 2^x(15/75)(52) = 3^x(364)$$

$$\frac{2^x}{3^x} = \left(\frac{2}{3}\right)^x = \frac{364}{819}$$

$$\frac{364}{819} = \frac{4}{9} = \left(\frac{2}{3}\right)^2 \Rightarrow \boxed{x = 2}$$

6)

$$((a-b)^2)^2((a+b)^2)^2 = (a-b)^4(a+b)^4$$

$$= (a^2-b^2)^4$$

$$= ((\sqrt{5\sqrt{6}-2} - \sqrt{\sqrt{6}+2})^2)^2$$

$$= (2\sqrt{6} - 2\sqrt{2})^2 = 32 - 16\sqrt{3}$$

$$= 16(2-\sqrt{3})$$

$$= t(2-\sqrt{3})$$

$$\Rightarrow \boxed{t = 16}$$

7)

$$a + \frac{1}{a} = A \qquad \Rightarrow (A+B)^2(A-B)^2 = (A^2-B^2)^2$$

$$\sqrt{3} = B$$

$$= \left(a^2 + \frac{1}{a^2}\right)^2 = \left(\sqrt{7-4\sqrt{3}} + \frac{1}{\sqrt{7-4\sqrt{3}}}\right)^2$$

aa

$$7-4\sqrt{3} = (2-\sqrt{3})^2$$

$$= \left(2-\sqrt{3} + \frac{1}{2-\sqrt{3}}\right)^2 = \left(\frac{(2-\sqrt{3})(2-\sqrt{3})+1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}\right)^2$$

$$= \left(\frac{2-\sqrt{3}+2+\sqrt{3}}{1}\right)^2 = (4)^2 = 16 = 2^t \Rightarrow \boxed{t = 4}$$

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8) $A = \sqrt[5]{2\sqrt[3]{112}} \quad \left(\frac{1}{2}\right)^{-\frac{2}{3}}$

$A = \sqrt[10]{2^{10}} \times \left(\frac{1}{2}\right)^{-\frac{2}{3}} = \sqrt[3]{2^3} \times 2^{\frac{2}{3}} = \sqrt[3]{2^3} \times \sqrt[3]{2^2}$

$= \sqrt[3]{2^6} = 4$

$\rightarrow (2A)^{-\frac{1}{3}} = (8)^{-\frac{1}{3}} = \sqrt[3]{\frac{1}{8}} = \sqrt[3]{2^{-3}} = 2^{-1} = \frac{1}{2}$

9)

$\sqrt[7]{a} = 27\,(a)^{\frac{15}{7}}$

$\rightarrow 27 = \frac{\sqrt[7]{a}}{\sqrt[7]{a^{15}}} = \sqrt[7]{a^{-14}} = 27 \Rightarrow a^{-2} = 27$

$a^2 = \frac{1}{27}$

$a^{-1} = \sqrt{27}$

$\left(\frac{1}{a} - 3\right) = a^{-1} - 3 = \sqrt{27} - 3 = 3\sqrt{3} - 3 = 3(\sqrt{3} - 1)$

$\frac{\left(\frac{1}{a} - 3\right)}{1+\sqrt{3}} = \frac{3(\sqrt{3}-1)}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3(4 - 2\sqrt{3})}{2} = \frac{3 \times 2(2-\sqrt{3})}{2}$

$= \boxed{3(2-\sqrt{3})}$

10)

$\sqrt{x+a} - \sqrt{x-4} = 2 \qquad \sqrt{x+a} + \sqrt{x-4} = B$

$(\sqrt{x+a} - \sqrt{x-4})(\sqrt{x+a} + \sqrt{x-4})$

$= x + a - x + 4 = a + 4 = 2B$

$\Rightarrow B = \frac{a+4}{2}$

$B - 2 = \frac{a+4-4}{2} = \frac{a}{2}$

$\sqrt{x-4} = \sqrt{x+a} - 2 \rightarrow B - 2 = 2\sqrt{x-4} = \frac{a}{2}$

$\Rightarrow a = 4\sqrt{x-4}$

PAPCO