

سوال ۱: رقم یکان ؟

(۲)

$$(1! + 3! + 5! + \dots + 25!) (2! + 4! + 6! + \dots + 24!) = 28$$

$\downarrow$ تعیین کتبه $\downarrow$ تعیین کتبه یکان $\downarrow$ تعیین کتبه دهی

$$1 + (5 \times 3 \times 1) = 7 \quad (2 \times 1) + (4 \times 3 \times 1) = 14 \quad 4 \times 7 = 28 \rightarrow 2 \text{ یکان} \checkmark$$

الف) $\sqrt[4]{4+\sqrt{4}} \times \sqrt{1+\sqrt{4}} = \sqrt[4]{\frac{4}{4+\sqrt{4}}} \times \sqrt[4]{1+\sqrt{4}} = \sqrt[4]{\frac{4(1+\sqrt{4})}{4+\sqrt{4}}} = \sqrt[4]{2} \checkmark$ سوال ۲: (۲)

ب) $\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2}\right) (\sqrt{5}-\sqrt{2} - \sqrt{2}+\sqrt{5}) = \left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2} \times \frac{\sqrt{10}-2}{\sqrt{10}-2}\right) (\sqrt{2}\sqrt{5} - \sqrt{2}+\sqrt{5}) = \frac{\sqrt{2}}{4} (\sqrt{2}\sqrt{5} - \sqrt{2}+\sqrt{5})$
 $= \frac{5}{4} = -1 \checkmark$

الف) $\frac{\sqrt{8}+\sqrt{2}}{5-\sqrt{4}} - 2(\sqrt[4]{9}-1)^{-1} = \left(\frac{\sqrt{8}+\sqrt{2}}{5-\sqrt{4}} \times \frac{5+\sqrt{4}}{5+\sqrt{4}}\right) - \left(\frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$ سوال ۳: (۲)
 $= \frac{19\sqrt{2}+19\sqrt{2}}{19} - \frac{2+2\sqrt{3}}{2} = \sqrt{3}+\sqrt{2} - 1 - \sqrt{3} = \sqrt{2}-1 \checkmark$

ب) $\frac{\sqrt{2}\sqrt{5}-1}{4+\sqrt{2}} + (\sqrt{5}-\sqrt{2})^{-1} = \left(\frac{\sqrt{2}\sqrt{5}-1}{4+\sqrt{2}} \times \frac{4-\sqrt{2}}{4-\sqrt{2}}\right) + \left(\frac{1}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}\right) = \frac{13\sqrt{2}-13}{13} + \frac{2+\sqrt{2}}{1}$
 $= -1 + \sqrt{2} + 2 + \sqrt{2} = 1 + 2\sqrt{2} \checkmark$

الف) $\frac{\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} \xrightarrow{\text{مضروب کنیم}} \frac{1+\sqrt{3} + \sqrt{3}-1 + 2\sqrt{3}-1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ سوال ۴: (۲)
 $= \frac{(\sqrt{4}+\sqrt{2})^2}{1} = \sqrt{4+\sqrt{4}} = \sqrt{4} \checkmark$

ب) $\sqrt[4]{28} \times \sqrt[4]{142} \times \sqrt[4]{2} = \sqrt[4]{28 \times 142 \times 2} = \sqrt[4]{2^8 \times 7 \times 11 \times 7} = 2 \times 7 \times 11 = 154 \checkmark$

$2^{28} + 2^{28+1} + 2^{28+2} + 2^{28+3} + 2^{28+4} + 2^{28+5} = 2^{28}(1+2+4+8+16+32) = 2^{28}(63)$ سوال ۵: (۲)
 $\frac{2^{28}(63)}{2^{28-1}(42)} = 2 \rightarrow 2^{28}(63) = \frac{1}{2} \times 2^{28} \times 42 \rightarrow 2^{28} \times \frac{63}{2} = 2^{28} \rightarrow 28 = 2 \checkmark$

$$(a^r + b^r + \sqrt{ab})^r (a^r + b^r - \sqrt{ab})^r = ((a+b)^r)^r ((a-b)^r)^r = (a^r - b^r)^{2r} \quad (r) \cdot (4)$$

$$= (\sqrt{4-5} - \sqrt{4+5})^r = (\sqrt{4-5} + \sqrt{4+5} - 2\sqrt{5})^r$$

$$\Rightarrow (2\sqrt{4-5} - \sqrt{5})^r = 5^r + 1 - 14\sqrt{5} = 14(5 - \sqrt{5}) = t(5 - \sqrt{5}) \rightarrow t = 14 \quad \checkmark$$

$$a = \sqrt[5]{5 - \sqrt{5}} = \sqrt[5]{5 - \sqrt{5}} \quad \left(a + \frac{1}{a} + \sqrt{5}\right)^r \left(a + \frac{1}{a} - \sqrt{5}\right)^r = \left(a + \frac{1}{a}\right)^{2r} - 5^r \quad (r) \cdot (V)$$

$$= \left(a^r + \frac{1}{a^r} + 5 - 5\right)^r \quad a^r + \frac{1}{a^r} + 5 = 5 - 5\sqrt{5} + \frac{1}{5 - 5\sqrt{5}} \times \frac{5 + 5\sqrt{5}}{5 + 5\sqrt{5}} + 5 = 5 - 5\sqrt{5} + 5 + 5\sqrt{5} + 5$$

$$= 14 = 5^t \rightarrow t = \frac{1}{5} \quad \checkmark$$

$$A = \sqrt[5]{5\sqrt{14}} \left(\frac{1}{5}\right)^{\frac{r}{5}} = \sqrt[5]{5} \times \sqrt[5]{14} \times \sqrt[5]{5^{-r}} = \sqrt[5]{5} \times \sqrt[5]{14} \times \sqrt[5]{5^{-r}} \quad (r) \cdot (A)$$

$$= \sqrt[5]{\frac{1}{5}} = \left(\frac{1}{5}\right)^{\frac{1}{5}} \quad \checkmark$$

$$\sqrt[5]{a} = 5\sqrt[5]{a^{15}} = \sqrt[5]{5^{15} \times a^{15}} \rightarrow a = 5^{15} \times a^{15} \rightarrow a^{14} = 5^{-15} \rightarrow a = \frac{1}{\sqrt[14]{5^{15}}} \quad (r) \cdot (9)$$

$$\frac{\sqrt{5^r} - 5}{1 + \sqrt{5}} \times \frac{1 - \sqrt{5}}{1 - \sqrt{5}} = \frac{-5(5 - \sqrt{5})}{-4} = \frac{5(5 - \sqrt{5})}{4} \quad \checkmark$$

$$\left. \begin{aligned} \sqrt{x+a} - \sqrt{x-5} &= 5 \\ \sqrt{x+a} + \sqrt{x-5} &= A \end{aligned} \right\} \begin{aligned} x+a - x+5 &= 5A = a+5 \\ A &= \frac{a+5}{5} \end{aligned} \quad (r) \cdot (10)$$

$$\sqrt{x+a} + \sqrt{x-5} - 5 = \frac{a+5}{5} - 5 = \left(\frac{a}{5}\right) \quad \checkmark$$