

سوال ۱: رقم یکان؟

$$(1! + 3! + 5! + \dots + 25!) (2! + 4! + 6! + \dots + 24!) = 28$$

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$$1 + (5 \times 2 \times 1) = 7 \quad (2 \times 1) + (4 \times 3 \times 2 \times 1) = 24 \quad 4 \times 7 = 28 \rightarrow 2 \text{ یکان}$$

الف) $\sqrt[4]{4+\sqrt{7}} \times \sqrt{1+\sqrt{7}} = \sqrt[4]{\frac{1}{4+\sqrt{7}}} \times \sqrt[4]{1+\sqrt{7}+2\sqrt{7}} = \sqrt[4]{\frac{2(4+\sqrt{7})}{4+\sqrt{7}}} = \sqrt[4]{2}$ سوال ۲:

ب) $\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2}\right) \left(\sqrt{5}-\sqrt{2}-\sqrt{2+\sqrt{5}}\right) = \left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2} \times \frac{\sqrt{10}-2}{\sqrt{10}-2}\right) \left(\sqrt{2}\sqrt{5}-\sqrt{2+\sqrt{5}}\right) = \frac{\sqrt{2}}{4} (\sqrt{2}\sqrt{5}-\sqrt{2+\sqrt{5}})$
 $= \frac{-2}{4} = -1$

الف) $\frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{4}} - 2(\sqrt[4]{9}-1)^{-1} = \left(\frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{4}} \times \frac{5+\sqrt{4}}{5+\sqrt{4}}\right) - \left(\frac{2}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$ سوال ۳:
 $= \frac{19\sqrt{2}+19\sqrt{3}}{19} - \frac{2+2\sqrt{3}}{2} = \sqrt{3}+\sqrt{2} - 1 - \sqrt{3} = \sqrt{2}-1$

ب) $\frac{\sqrt{27}-1}{4+\sqrt{3}} + (\sqrt{3}-\sqrt{2})^{-1} = \left(\frac{\sqrt{27}-1}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}}\right) + \left(\frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right) = \frac{13\sqrt{3}-13}{13} + \frac{2+\sqrt{6}}{1}$
 $= -1 + \sqrt{3} + 2 + \sqrt{6} = 1 + \sqrt{3} + \sqrt{6}$

الف) $\frac{\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} \xrightarrow{\text{مضروب کردن}} \frac{1+\sqrt{3}+\sqrt{3}-1+2\sqrt{3}-1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ سوال ۴:
 $= \frac{(\sqrt{4}+\sqrt{2})^2}{1} = \sqrt{4+\sqrt{4}} = \sqrt{4} = 2$

ب) $\sqrt[4]{28} \times \sqrt[4]{142} \times \sqrt[4]{2} = \sqrt[4]{28 \times 142 \times 2} = \sqrt[4]{2^4 \times 7 \times 11 \times 7 \times 11 \times 2} = \sqrt[4]{2^6 \times 7^2 \times 11^2} = 2 \times 7 \times 11 = 154$

سوال ۵: $\frac{2^{28} + 2^{28+1} + 2^{28+2} + 2^{28+3} + 2^{28+4} + 2^{28+5}}{2^{28-2} + 2^{28-1} + 2^{28} + 2^{28+1} + 2^{28+2} + 2^{28+3}} = \frac{2^{28}(1+2+4+8+16+32)}{2^{28-2}(1+2+4+8+16+32)} = \frac{2^{28}(63)}{2^{26}(63)} = 2^2 = 4$
 $\frac{2^{28}(63)}{2^{28-1}(63)} = 2 \rightarrow 2^{28}(63) = \frac{1}{2} \times 2^{28} \times 2^2(63) \rightarrow 2^{28} \times \frac{1}{2} = 2^{27} \rightarrow 27 = 28$

$$\begin{aligned}
 (a^r + b^r + rab)(a^r + b^r - rab) &= ((a+b)^r)((a-b)^r) = (a^r - b^r)^r \quad (4) \\
 &= (\sqrt{4-r} - \sqrt{4+r})^r = (\sqrt{4-r} + \sqrt{4+r} - 2\sqrt{r})^r \\
 \Rightarrow (2\sqrt{4-r} - 2\sqrt{r})^r &= r^r + 1 - 14\sqrt{r} = 14(r - \sqrt{r}) = t(r - \sqrt{r}) \rightarrow \boxed{t=14}
 \end{aligned}$$

$$\begin{aligned}
 a &= \sqrt[r]{V - r\sqrt{r}} = \sqrt[r]{r^r - r\sqrt{r}} \quad \left(a + \frac{1}{a} + \sqrt{r}\right)^r \left(a + \frac{1}{a} - \sqrt{r}\right)^r = \left(a + \frac{1}{a}\right)^r - r^r \quad (V) \\
 &= \left(a^r + \frac{1}{a^r} + r - r\right)^r \quad a^r + \frac{1}{a^r} + r = V - r\sqrt{r} + \frac{1}{V - r\sqrt{r}} \times \frac{V + r\sqrt{r}}{V + r\sqrt{r}} + r = V - r\sqrt{r} + V + r\sqrt{r} + r \\
 &= 14 = r^t \rightarrow \boxed{t=r}
 \end{aligned}$$

$$\begin{aligned}
 A &= \sqrt[r]{r^r \sqrt{14}} \left(\frac{1}{r}\right)^{r \frac{r}{r}} = \sqrt[r]{r^{10}} \times \sqrt[r]{r^r} = r^r \quad (rA)^{\frac{1}{r}} = (r^r)^{-\frac{1}{r}} = \left(\frac{1}{r^r}\right)^{\frac{1}{r}} \quad (A) \\
 &= \sqrt[r]{\frac{1}{r^r}} = \boxed{\frac{1}{r}}
 \end{aligned}$$

$$\sqrt[r]{a} = rV \times \sqrt[r]{a^{10}} = \sqrt[r]{r^{r1} \times a^{10}} \rightarrow a = r^{r1} \times a^{10} \rightarrow a^{1r} = r^{-r1} \rightarrow a = \frac{1}{\sqrt[r]{r^r}} \quad (9)$$

$$\frac{\sqrt{r^r} - r}{1 + \sqrt{r}} \times \frac{1 - \sqrt{r}}{1 - \sqrt{r}} = \frac{-r(4 - r\sqrt{r})}{-r} = \boxed{r(r - \sqrt{r})}$$

$$\begin{aligned}
 \left. \begin{aligned} \sqrt{K+a} - \sqrt{K-r} &= r \\ \sqrt{K+a} + \sqrt{K-r} &= A \end{aligned} \right\} K+a - K+r &= rA = a+r \\
 \rightarrow A &= \frac{a+r}{r} \quad (10) \\
 \sqrt{K+a} + \sqrt{K-r} - r &= \frac{a+r}{r} - r = \boxed{\frac{a}{r}}
 \end{aligned}$$